Time-varying group formation-tracking control for general linear multi-agent systems with switching topologies and time-varying delays

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Abstract— **Time-varying group formation-tracking control for general linear multi-agent systems with switching topologies and varying time delays is studied in this paper. Due to the deferent effects in coordinated problem, the agents in group formation-tracking are divided into two roles, leaders and followers, respectively. The followers are allowed to achieve the expected subgroup formation and, in the meantime, tracking the trajectory of the leaders in each group. Firstly, utilizing the neighboring information, the observers is proposed for each follower to estimate the leader's state in the subgroup. Based on the transformed of the estimated error and Lyapunov theory, the effectiveness of the proposed observer is proven. Secondly, by incorporating the state observer in the formation-tracking protocol, the novel controller is put forward to solve the group formation-tracking problem under the influence of both time-varying delays and switching networks. Then, an algorithm to determine the gain matrix is presented, and the convergence the of group formation error is also demonstrated. Finally, a numerical simulation result is given to verify the practical of the theoretical results.**

Index Terms— **Group formation-tracking problem; general linear multi-agent systems; time-varying delays; switching topologies**

I. Introduction

In recent years, coordinated control has been applied in varies engineering areas such as unmanned aerial vehicles [1], [2], spacecraft [3], [4], and underactuated surface vessels [5]. As an important method in coordinated problem, consensus control has aroused considerable research interest due to the low computing consumption and theoretical significance. Consensus-based control of multi-agent systems (MASs) can be divided into different categories, for instant, formation control [6], containment control [7], formation-tracking control [8], [9], etc.

As one of the most attracting areas, formation control aiming a group of agents to perform specific formation are introduced. Based on consensus problem, some formation control problems for first-order and second-order were studied [10]. Moreover, Dong et al. in [11] proposed a formation feasibility condition for the desire formation, which means not all formations were allowed to achieve. However, in many complicated missions such as patrol or detection, the agents should not only realize the desire formation, but also track the trajectory generated by the virtual or real leaders. Therefore, the research on formation-tracking control has received much attention, and a large amount of results have been reported [12], [13]. Due to the congestion of the interaction channel, the interaction topologies can be switching. Therefore, time-varying formation tracking control for MASs with switching interaction networks is taken into consideration. Yu in [14] investigated time-varying formation tracking problem under the circumstance of both switching interactions and leader's unknown input. As an important factor in the control system, communication delays can effect the performance of the agents and cause the divergence. Hence, a consensus-based protocol was put forward to solve a class of MASs with time-varying delays in [15]. The above mentioned researches only focus on a single formation problem. In practical applications, for instance, multiple patrolling or targets enclosing. The MASs should be divided into several group and play different role in the mission simultaneously. An acyclic partition of the nodes was used to solve the group formation problem in the work of Qin et al. [16]. An observed-based controller is put forward for general linear systems to achieve group formation tracking in [17]. Han et al. in [18] investigated the group formation-tracking problem for second-order systems with time-varying communication delays. Moreover, considering the switching interaction topologies, a protocol was put forward to realize the group formation-tracking problem based on neighboring information in [19].

The above mentioned investigations considered the time-varying delay, switching topology, and unknown input, etc. But none of them study the group formation-tracking problem with both communication delays and switching networks. Considering both constraints at the same time are much more difficult than considering one case separately. To the best of our knowledge, the group formation-tracking problems with both time-varying delays and switching interaction topologies are still open.

Motivated by the above investigations. This paper studies the group formation-tracking for general linear MASs with both communication delays and switching topologies.

1) Compared with the works in [10], [11], this paper focuses on group formation tracking problem, in which all the followers can be divided into several groups to execute the different tasks.
2) Note that the previous works on group formation-tracking in [14], [15] only study the second-order MASs. However, to extending the control approach,

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generic MASs are studied in this investigation, and both first-order or second-order systems can be regarded as a special case in general system.

3) Both communication delays and switching topologies are studied in this paper, which means the group formation-tracking can accommodate in a more complex environment. Hence, the group formation-tracking problems in [14], [15] are restrictive.

The organization of the rest paper is given as followings. Section II demonstrates some basic concepts on graph theory and some significant definitions are also proposed. In section III, a state observer and group formation-tracking protocol are put forward for MAS with communication delays and switching networks. Moreover, a feasible algorithm is also presented. Section IV shows a numerical simulation for the mentioned method. Section V concludes the whole works.

Throughout this paper, let $\otimes$ denote the Kronecker product of two matrices. I represents an identify matrix with appropriate size. Let $\mathbf{1}$ and 0_n be a zero matrices with dimension a column vectors with 1 and n as its element.

II. Preliminaries and problem description

In this section, some basic concepts and notations on graph theory and the problem descriptions are demonstrated.

A. Preliminaries

A weighted undirected graph G can be represented by $\{V,T,W\}$, where $V = \{v_1, v_2, \cdots v_N\}$ is a set of nodes, $T \subseteq \{(v_i,v_j) : v_i, v_j \in V\}$ is the set of edges, and $W = [a_{ij}] \in \mathbb{R}^{N\times N}$ is a weighted adjacency matrix. Let $e_{ij} = (v_i, v_j)$ denote the edge of G and w_{ij} denote the nonnegative element with e_{ji}. Define $w_{ij} > 0$ if and only if $e_{ji} \in T$ and $w_{ij} = 0$ otherwise. $N_i = \{v_j \in V : (v_j, v_i) \in T\}$ is the set of neighbors of node v_i. The Laplacian matrix L is defined as $L = D - W$, where $D = diag\left\{\sum_{j=1}^{N} w_{1j}, \sum_{j=1}^{N} w_{2j}, \cdots, \sum_{j=1}^{N} w_{Nj}\right\}$. A path from node v_{i1} to v_{ik} is a series of ordered edges (v_{i1}, v_{i2}), (v_{i2}, v_{i3}), $\cdots$, (v_{ik-1}, v_{ik}). The definition of undirected graph is that $v_{ij} \in T$ implies $v_{ji} \in T$ and $w_{ij} = w_{ji}$. The undirected graph is said to be connected if there is a path between any distinct pair of nodes.

It is assumed that the interaction topologies are switching. Let $[t_k, t_{k+1})$ $(k \in \mathbb{N})$ denotes an infinite sequence of uniformly bounded non-overlapping time intervals with $t_0 = 0$, $t_k - t_{k+1} \geqslant T_d > 0$. T_d is said as the dwell time, during which the graph keeps fixed. The graph changes at switching sequence t_{k+1}. Let $\sigma(t) : [0, \infty) \to \{1, 2, \cdots, h\}$ denotes a switching signal. $G_{\sigma(t)}$ and $L_{\sigma(t)}$ represent the graph and Laplacian matrix at t. Let $L^F_{\sigma(t)}$ and $L^F_{\sigma(t)}$ denote Laplacian matrix among the followers and leaders.

Definition 1: An agent is called a leader if its neighbor set has no agent, otherwise it is called a follower if it has at least one neighbor.

Lemma 1: If G is connected, then L has a simple 0 eigenvalue with $1_N \Big/ \sqrt{N}$ as its right eigenvector, and all the other eigenvalues are positive.

B. Problem description

Consider a MAS with M leaders and N followers and the system is divided into several groups. Let $E = \{1, 2, \cdots, M\}$ and $F = \{M + 1, M + 2, \cdots, M + N\}$ denote the leaders set and followers set, respectively. The target of the group formation-tracking is that the followers should form the desired sub-formation and track the trajectory of each group leader in the meanwhile.

It is assumed that the MAS has $g \in \mathbb{N}\,(g \geqslant 1)$ subgroups and the separation of the nodes for the followers V_F are defined as $V_1, V_2, \cdots, V_g$, which satisfies $V_k \neq \emptyset\,(k = 1, 2, \cdots, g)$, $\cup_{k=1}^{g} V_k = V_F$ and $V_k \cap V_m = \emptyset$ $(k, m \in \{1, 2, \cdots, g\}\,; k \neq m)$. For follower $i, j \in F$, define $\bar{i}$ and $\bar{j}$ as the index of the subgroups to which agents i, j belong. If $\bar{i} = \bar{j}$, the followers i and j are said to be in the same subgroup. The number of the followers and leaders in subgroup $\bar{i}\,(\bar{i} \in 1, 2, \cdots, g)$ are denoted by $n_{\bar{i}}$ and $n_{\bar{i}l}$, respectively. Note that there exists only one leader in each group, therefore, $\sum\limits_{\bar{i}=1}^{g} n_{\bar{i}} = N$ and $\sum\limits_{\bar{i}=1}^{g} n_{\bar{i}l} = M$. Let $V_{\bar{i}} = \{\Xi_{\bar{i}} + 1, \Xi_{\bar{i}} + 2, \cdots, \Xi_{\bar{i}} + n_{\bar{i}}\}$, $\Xi_{\bar{i}} = \sum\limits_{k=1}^{\bar{i}-1} n_k$ denote the index of subgroup $\bar{i}$.

The leader of subgroup $\bar{i}\,(\bar{i} = \{1, 2, \cdots, g\})$ is defined as

$$\dot{z}_0^{\bar{i}}(t) = Az_0^{\bar{i}}(t) \tag{1}$$

The follower of $i\,(i \in \{\Xi_{\bar{i}} + 1, \Xi_{\bar{i}} + 2, \cdots, \Xi_{\bar{i}} + n_{\bar{i}}\})$ in subgroup $\bar{i}\,(\bar{i} = \{1, 2, \cdots, g\})$ can be modeled by

$$\dot{x}_i^{\bar{i}}(t) = Ax_i^{\bar{i}}(t) + Bu_i^{\bar{i}}(t) \tag{2}$$

where $x_i^{\bar{i}}(t) \in \mathbb{R}^n$ and $u_i^{\bar{i}}(t) \in \mathbb{R}^m$ are the state, control input of ith follower. $z_0^{\bar{i}}(t) \in \mathbb{R}^n$ is the state of the jth leader. $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are the constant gain known matrices with rank$(B) = m$. The system matrixes (A, B) is stabilizable.

Assumption 1: The $\{V_1, V_2, \cdots, V_g\}$ is an acyclic partition of the node set V_F and for each subgroup $\bar{i}\,(\bar{i} \in 1, 2, \cdots, g)$, the corresponding topology $\bar{G}_{\sigma(t)\bar{i}}$ of the followers is undirected and connected.

The Laplacian matrix $L_{\sigma(t)}$ of the group MASs is shown as follows:

$$\begin{bmatrix} 0_{M\times M} & 0_{M\times N} \\ L^{EF}_{\sigma(t)} & L^{F}_{\sigma(t)} \end{bmatrix}$$

Based on Assumption 1, $L^{EF}_{\sigma(t)}$ and $L^{F}_{\sigma(t)}$ has the following form

$$L^{EF}_{\sigma(t)} = \begin{bmatrix} L^{EF}_{\sigma(t)1} & 0 & \cdots & 0 \\ 0 & L^{EF}_{\sigma(t)2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & L^{EF}_{\sigma(t)g} \end{bmatrix} \tag{3}$$

$$L^{F}_{\sigma(t)} = \begin{bmatrix} L^{F}_{\sigma(t)1} & 0 & \cdots & 0 \\ L^{F}_{\sigma(t)12} & L^{F}_{\sigma(t)2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ L^{F}_{\sigma(t)g1} & L^{F}_{\sigma(t)g2} & \cdots & L_{\sigma(t)g} \end{bmatrix} \tag{4}$$

Where $L^{EF}_{\sigma(t)} \in \mathbb{R}^{N\times M}$ and $L^{F}_{\sigma(t)} \in \mathbb{R}^{N\times N}$ denote to the interaction between the leaders to followers and among the followers. Let $L^{EF}_{\sigma(t)\bar{i}}$ represents the communication between the leader and the followers of the subgroup $\bar{i}$, and $L^{F}_{\sigma(t)\bar{i}\bar{j}}$ represents the followers interaction from subgroup $\bar{i}$ to subgroup $\bar{j}$ $(\bar{i},\bar{j} \in \{1,2,\cdots,g\})$.

Assumption 2: For any communication topologies, the sum of each row of $L^{F}_{\sigma(t)\bar{i}\bar{j}}$, $\bar{i},\bar{j} \in \{1,2,\cdots,g\}$ is equal to zeros. And the eigenvalues of the $L^{F}_{\sigma(t)} \in \mathbb{R}^{N\times N}$ are different.

Lemma 2: Based on Assumption 1 and Assumption 2, the Laplacian matrix $L_{\sigma(t)}$ has $g-th$ 0 eigenvalues with $u_1 = \left[1, 0^T_{M-1}, 1^T_{n_1}, \cdots 0^T_{N-n_1}\right]^T$, $u_2 = \left[0, 1, 0^T_{M-2}, 0^T_{n_1}, 1^T_{n_2}, \cdots, 0^T_{N-n_1-n_2}\right]^T$, $\cdots$, $u_g = \left[0^T_{M-1}, 1, 0^T_{N-n_g}, 1^T_{n_g}\right]^T$ as the corresponding right eigenvectors, and all the left $N-g$ eigenvalues have the positive real parts under the influence of both time-varying delays and switching interaction topologies.

Let $h^{\bar{i}}(t) = \left[h^T_{\Xi_{\bar{i}}+1}(t), h^T_{\Xi_{\bar{i}}+2}(t), \cdots, h^T_{\Xi_{\bar{i}}+n_{\bar{i}}}(t)\right]^T \in \mathbb{R}^{nn_i}$ represent the expected formation of subgroup $\bar{i}$ $(\bar{i} \in \{1,2,\cdots,g\})$, where each component of $h^T_{\Xi_{\bar{i}}+j}(t)$ $(j \in \{\Xi_{\bar{i}}+1, \Xi_{\bar{i}}+2, \cdots, \Xi_{\bar{i}}+n_{\bar{i}}\})$ is piecewise continuously differentiable. Denote $x^{\bar{i}}(t) = \left[x^{\bar{i}T}_{\Xi_{\bar{i}}+1}(t), x^{\bar{i}T}_{\Xi_{\bar{i}}+2}(t), \cdots, x^{\bar{i}T}_{\Xi_{\bar{i}}+n_{\bar{i}}}(t)\right]^T \in \mathbb{R}^{nn_{\bar{i}}}$ for the followers of subgroup $\bar{i}$ $\bar{i} \in \{1,2,\cdots,g\}$.

Definition 2: If for any given bounded initial values, the MAS (1) and (2) is said to achieve the group formation tracking for any subgroup $\bar{i}$ $(\bar{i} \in \{1,2,\cdots,g\})$

$$\lim_{t\to 0}\left(x^{\bar{i}}(t) - h^{\bar{i}}(t) - \left(1_{n_i} \otimes z^{\bar{i}}_0(t)\right)\right) = 0 \tag{5}$$

The rest of this paper will concentrate on

1) Under what condition the MAS (1) and (2) can realized the desired group formation tracking.
2) How to design the state observer and group formation tracking protocol under the influence of time-varying delays and switching interaction topologies.

III. Main results

In this section, state observer for each follower to estimate the group leader is introduced under the influence of both time-varying communication delays and switching interaction topologies. Then, the effectiveness of the proposed observer is to be proved. Furthermore, an observer-based group formation protocol is proposed and an algorithm to determine the constant matrix in the protocol is also put forward.

Consider the following state observer for each follower:

$$\begin{aligned}\dot{\hat{\zeta}}^{\bar{i}}_i(t) = A\hat{\zeta}^{\bar{i}}_i(t) - K_1 \Bigg(&\sum_{j\in N^i_{\sigma(t)}, c=\{1,2,\cdots,g\}} w_{ij}\left(\hat{\zeta}^{\bar{i}}_i(t-\tau(t))\right.\\ &\left.-\hat{\zeta}^{c}_j(t-\tau(t))\right) + w_{i0}\left(\hat{\zeta}^{\bar{i}}_i(t-\tau(t)) - z^{\bar{i}}_0(t-\tau(t))\right)\Bigg)\end{aligned} \tag{6}$$

Where $\hat{\zeta}^{\bar{i}}_i(t)$ is ith follower' observer belongs to the subgroup $\bar{i}$ $(\bar{i} \in \{1,2,\cdots,g\})$ and $z^{\bar{i}}_0(t)$ is the leader of subgroup $\bar{i}$. $\tau(t)$ represents the time-varying communication delays. K_1 is the constant gain matrix can be calculated by the later Algorithm.

Assumption 3: The time-varying communication delays $\tau(t)$ satisfies $0 \leqslant \tau(t) \leqslant \sigma$ and $|\dot{\tau}(t)| \leqslant \delta < 1$. Note that σ and δ are known constants, which means $\tau(t)$ is bounded.

The following lemmas are presented to prove the effectiveness of the proposed observer.

Lemma 3: A vector-valued function is denoted by $\eta(t) \in \mathbb{R}^{2d}$, which entries are first-order continuous-derivative. One gets

$$\begin{aligned}&-\int_{t-\tau(t)}^{t} \dot{\eta}^T(s) P \dot{\eta}(s) ds\\ &\leqslant \gamma^T(t)\begin{bmatrix} X^T_1 + X_1 & -X^T_1 + X_2 \\ * & -X^T_2 - X_1\end{bmatrix}\gamma(t)\\ &\quad + \tau(t)\gamma^T(t)\begin{bmatrix} X^T_1 \\ X^T_2\end{bmatrix} P^{-1}[X_1, X_2]\gamma(t)\end{aligned} \tag{7}$$

where $X_1, X_2 \in \mathbb{R}^{2d}$, $\gamma(t) = \left[\eta^T(t), \eta^T(t-\tau(t))\right]^T$ and P is a positive definite symmetric matrix.

Let $\bar{\lambda}_1 = \min\left\{\lambda^i_{\sigma(t)}\right\}$, $\bar{\lambda}_2 = \max\left\{\lambda^i_{\sigma(t)}\right\}$, $\sigma(t) \in \{1,2,\cdots,p\}$, $\lambda^i_{\sigma(t)}$ is the eigenvalue of real symmetric positive definite matrix.

Lemma 4: For any i, the switching signal $\sigma(t) \in \{1,2,\cdots,p\}$, $\Theta^i_{\sigma(t)} = \Phi_0 + \lambda^i_{\sigma(t)}\Phi_1 < 0$ if and only if $\Theta_i = \Phi_0 + \bar{\lambda}_i\Phi_1 < 0$ $(i \in \{1,2\})$.

Lemma 5: Let $\hat{\zeta}^{\bar{i}}(t) = \left[\hat{\zeta}^{\bar{i}T}_{\Xi_{\bar{i}}+1}(t), \hat{\zeta}^{\bar{i}T}_{\Xi_{\bar{i}}+2}(t), \cdots, \hat{\zeta}^{\bar{i}T}_{\Xi_{\bar{i}}+n_{\bar{i}}}(t)\right]^T \in \mathbb{R}^{nn_{\bar{i}}}$, If the state observer satisfies the following equation

$$\lim_{t\to 0}\left(\hat{\zeta}^{\bar{i}}(t) - (1_{n_i} \otimes I_n) z^{\bar{i}}_0(t)\right) = 0 \tag{8}$$

Then the observer is said to estimate the leader's state for the subgroup $\bar{i}$.

The constant matrix in state observer K_1 is designed as the following linear matrix inequality (LMI) If there exist positive symmetric matrices R, Ω, X and real matrix $\bar{K}_1$, LMI (9) is feasible for any $\bar{\lambda}^F_i$ $(i = 1,2)$

$$\prod(\bar{\lambda}_i) = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & 0 & R \\ * & \Xi_{22} & \Xi_{23} & \sigma X & 0 \\ * & * & -\sigma X & 0 & 0 \\ * & * & * & -\sigma X & 0 \\ * & * & * & * & -\Omega \end{bmatrix} < 0 \tag{9}$$

where

$$\begin{aligned}\Xi_{11} &= RS^T + SR - \bar{\lambda}^F_i\bar{K}_1 - \bar{\lambda}^F_i\bar{K}^T_1 - (1-\delta)\Omega\\ \Xi_{12} &= R - \bar{\lambda}^F_i\bar{K}_1 - (2-\delta)\Omega\\ \Xi_{13} &= \sigma RS^T - \sigma\bar{\lambda}^F_i\bar{K}^T_1\\ \Xi_{22} &= -(3-\delta)\Omega\\ \Xi_{23} &= -\sigma\bar{\lambda}^F_i\bar{K}^T_1\end{aligned}$$

$\bar{\lambda}^F_i$ $(i = 1,2)$ are the minimum and maximum eigenvalues of the followers Laplacian matrix $L^F_{\sigma(t)}$.

Then the gain matric can be defined as $K_1 = \bar{K}_1 \Omega^{-1}$.

Based on the calculated K_1 and Theorem 1 the following Theorem can be derived.

Theorem 1: The proposed state observer can estimate the leader state for each subgroup under the influence of both time-varying communication delays and switching interaction topologies.

Proof: Form Theorem 1, $\tilde{\zeta}(t)$ denotes the estimate error. one gets

$$\begin{aligned} \dot{\tilde{\theta}}(t) &= (I_{M+N} \otimes A)\tilde{\theta}(t) \\ &- \left(\begin{bmatrix} 0 & 0 \\ 0 & L^F_{\sigma(t)} \end{bmatrix} \otimes K_1 \right) \tilde{\theta}(t-\tau(t)) \end{aligned} \tag{10}$$

Note that $\tilde{\theta}(t) = \left[z^T(t), \tilde{\zeta}^T(t) \right]^T$, which means

$$\begin{cases} \dot{z}(t) = (I_M \otimes A) z(t) \\ \dot{\tilde{\zeta}}(t) = (I_N \otimes A)\tilde{\zeta}(t) - \left(L^F_{\sigma(t)} \otimes K_1 \right) \tilde{\zeta}(t-\tau(t)) \end{cases} \tag{11}$$

Considering the following common Lyapunov-Krasovskii candidate function:

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{12}$$

where
$V_1(t) = \tilde{\zeta}^T(t)\left(I_N \otimes R^{-1}\right)\tilde{\zeta}(t)$,
$V_2(t) = \int_{t-\tau(t)}^{t} \tilde{\zeta}^T(s)\left(I_N \otimes \Omega^{-1}\right)\tilde{\zeta}(s)\,ds$,
$V_3(t) = \int_{-\sigma}^{0}\int_{t+\mu}^{t} \dot{\tilde{\zeta}}^T(s)\left(I_N \otimes X^{-1}\right)\dot{\tilde{\zeta}}(s)\,ds\,d\mu$.

It can be verified that $L^F_{\sigma(t)}$ is positive symmetric matrices symmetric, Let $\Lambda^F_{\sigma(t)} = diag\left(\lambda^1_{\sigma(t)}, \lambda^2_{\sigma(t)}, \cdots, \lambda^N_{\sigma(t)}\right)$, then there exists an orthogonal matrix $M_{\sigma(t)} \in \mathbb{R}^{N\times N}$ satisfying $M^T_{\sigma(t)} L^F_{\sigma(t)} M_{\sigma(t)} = \Lambda^F_{\sigma(t)}$.

Define $\eta(t) = \left(M^T_{\sigma(t)} \otimes I_{Nn}\right)\tilde{\zeta}(t) = \left[\eta_1^T(t), \eta_2^T(t), , \cdots, \eta_N^T(t)\right]^T$, take the derivative of $V(t)$ along the (12)

$$\begin{aligned} &\dot{V}_1(t) = \\ &\sum_{i=1}^{N} \hat{\eta}_i^T(t) \begin{bmatrix} R^{-1}S + S^T R^{-1} & -\lambda^i_{\sigma(t)} R^{-1} B K_1 \\ * & 0 \end{bmatrix} \hat{\eta}_i(t) \end{aligned} \tag{13}$$

where $\hat{\eta}_i(t) = \left[\eta_i^T(t), \eta_i^T(t-\tau(t))\right]^T$.

Based on Assumption 3, $\dot{V}_2(t)$ can be written as

$$\begin{aligned} &\dot{V}_2(t) \le \eta^T(t)\left(I_N \otimes \Omega^{-1}\right)\eta(t) \\ &- (1-\delta)\,\eta^T(t-\tau(t))\left(I_N \otimes \Omega^{-1}\right)\eta(t-\tau(t)) \\ &= \sum_{i=1}^{N} \hat{\eta}_i^T(t) \begin{bmatrix} \Omega^{-1} & 0 \\ 0 & -(1-\delta)\Omega^{-1} \end{bmatrix} \hat{\eta}_i(t) \end{aligned} \tag{14}$$

$$\begin{aligned} \dot{V}_3(t) &= \sigma \dot{\eta}^T(t)\left(I_N \otimes X^{-1}\right)\dot{\eta}(t) \\ &- \int_{t-\sigma}^{t} \dot{\eta}^T(s)\left(I_N \otimes X^{-1}\right)\dot{\eta}(s)\,ds \end{aligned} \tag{15}$$

Let $\varpi_i = \left[S\ ,\ -\lambda^i_{\sigma(t)} B K_1\right]$, the first half of the equation (15) is given as

$$\sigma \dot{\eta}^T(t)\left(I_N \otimes X^{-1}\right)\dot{\eta}(t) = \sigma \sum_{i=1}^{N} \hat{\eta}_i^T(t)\,\varpi_i^T X^{-1} \varpi_i \hat{\eta}_i(t) \tag{16}$$

From Assumption 4 and Lemma 2. The latter part is given as

$$\begin{aligned} &- \int_{t-\sigma}^{t} \dot{\eta}^T(s)\left(I_N \otimes X^{-1}\right)\dot{\eta}(s)\,ds \\ &\le - \int_{t-\tau(t)}^{t} \dot{\eta}^T(s)\left(I_N \otimes X^{-1}\right)\dot{\eta}(s)\,ds \\ &= \sum_{i=1}^{N} \left(- \int_{t-\tau(t)}^{t} \eta_i^T(t) X^{-1} \eta_i(t)\,ds \right) \\ &\le \sum_{i=1}^{N} \hat{\eta}_i^T(t) \left(\begin{bmatrix} M_1^T + M_1 & -M_1^T + M_2 \\ * & -M_2^T - M_2 \end{bmatrix} \right. \\ &\left. + \sigma \begin{bmatrix} M_1^T \\ M_2^T \end{bmatrix} X^{-1} [M_1, M_2] \right) \hat{\eta}_i(t) \end{aligned} \tag{17}$$

Define $M_1 = -R^{-1}, M_2 = \Omega^{-1}$, form (12) to (17), ones get

$$\dot{V}(t) \leqslant \sum_{i=1}^{N} \hat{\eta}_i^T(t) \mathrm{Z}_i \hat{\eta}_i(t) \tag{18}$$

where

$$\mathrm{Z}_i = \mathrm{T}_i + \sigma \varpi_i^T X^{-1} \varpi_i + \sigma \begin{bmatrix} -R^{-T} \\ \Omega^{-T} \end{bmatrix} X^{-1} \left[-R^{-1}, \Omega^{-1}\right],$$
$$\mathrm{T}_i = \begin{bmatrix} \mathrm{T}_{i11} & \Omega^{-1} + R^{-1} - \lambda^i_{\sigma(t)} R^{-1} B K_1 \\ * & -(3-\delta)\,\Omega^{-1} \end{bmatrix},$$
$$\mathrm{T}_{i11} = -2R^{-1} + R^{-1}S + S^T R^{-1} + \Omega^{-1}.$$

It can be verified by Schur complement lemma, $\mathrm{Z}_i < 0$ is equivalent to $\Psi_i < 0$

$$\Psi_i = \begin{bmatrix} T_i & \sigma \varpi_i^T & \sigma \begin{bmatrix} -R^{-1} & -\Omega^{-1} \end{bmatrix} \\ * & \sigma S^{-1} & 0 \\ * & * & -\sigma S^{-1} \end{bmatrix} < 0$$

Choosing $\Gamma = \begin{bmatrix} R & 0 \\ \Omega & \Omega \end{bmatrix}$ and $\bar{\Gamma} = diag\{T, I, X\}$, then one gets

$$\bar{\Gamma}^T \psi_i \bar{\Gamma} = \begin{bmatrix} \Gamma^T T \Gamma_i & \sigma \Gamma^T \varpi_i^T & \sigma \begin{bmatrix} 0 & X \end{bmatrix} \\ * & \sigma X & 0 \\ * & * & -\sigma X \end{bmatrix}$$

Based on the calculated $K_1 = \bar{K}_1 \Omega^{-1}$ and Lemma 3, $\prod\left(\bar{\lambda}_i\right) < 0$ are equivalent to $\prod\left(\lambda^i_{\sigma(t)}\right) < 0$ $(i = 2, 3, \cdots, N, \sigma(t) = 1, 2, \cdots, p)$. Then according to Schur complement lemma, $\prod\left(\lambda^i_{\sigma(t)}\right) < 0$ if and only if $\bar{\Gamma}^T \psi_i \bar{\Gamma} < 0$. One gets

$$\lim_{t\to\infty} \tilde{\zeta}(t) = 0 \tag{19}$$

Therefore observer's error $\tilde{\zeta}(t)$ converges to zero as $t \to \infty$ with both communication delays and switching interaction topologies. This completes the proof.

■

Consider the following observer-based group formation-tracking protocol for follower i $(i \in \{\Xi_{\bar{i}} + 1, \Xi_{\bar{i}} + 2, \cdots, \Xi_{\bar{i}} + n_{\bar{i}}\})$ in subgroup $\bar{i}$ $(\bar{i} = \{1, 2, \cdots, g\})$

$$u_i^{\bar{i}}(t) = K_2 x_i^{\bar{i}}(t) + K_3\left(\hat{\zeta}_i^{\bar{i}}(t) + h_i^{\bar{i}}(t)\right) + r_i^{\bar{i}}(t) \tag{20}$$

where K_2 and K_3 are the constant matrix, $r_i^{\bar{i}}(t) \in \mathbb{R}^m$ is the compensation input for the group formation tracking.

It is verified that $\mathrm{rank}(B) = m$, Let $\Gamma = \left[B_1^T, B_2^T\right]^T$ with $B_1 \in \mathbb{R}^{(n-m)\times n}$ and $B_2 \in \mathbb{R}^{(n-m)\times n}$. $B_2B = 0_{(n-m)\times m}$ and $B_1B = I_{(n-m)\times m}$.

Algorithm 1. Steps of designing the gain matrix

Step 1: For the given formation, check the following formation feasibility condition.

$$\lim_{t\to\infty}\left(B_2Ah_i^{\bar{i}}(t) - B_2\dot{h}_i^{\bar{i}}(t)\right) = 0 \tag{21}$$

If (21) is feasible, then continue; otherwise, the Algorithm stops, choose another formation.

Step 2: The formation compensation input $r_i^{\bar{i}}(t)$ is defined as

$$r_i^{\bar{i}}(t) = B_1\dot{h}_i^{\bar{i}}(t) - (B_1A)\,h_i^{\bar{i}}(t) \tag{22}$$

Step 3: Choose suitable K_1 for state observer $\hat{\zeta}_i^{\bar{i}}(t)$ under the influence of time-varying delays and switching interaction topologies.

Step 4: The gain matrix K_2 is design to make $A + BK_2$ is Hurwitz, and $K_3 = -K_2$.

Theorem 2: If there exists a formation satisfying (21), MASs (2) and (1) are said to achieve the group formation tracking with time-varying delays and switching interaction topologies under the protocol designed by Algorithm 1.

Proof: Based on protocol (20), the followers systems can be written as

$$\dot{x}_i^{\bar{i}}(t) = (A + BK_2)\,x_i^{\bar{i}}(t) + BK_3\hat{\zeta}_i^{\bar{i}}(t) + BK_3h_i^{\bar{i}}(t) + Br_i^{\bar{i}}(t) \tag{23}$$

Define observer error $e_i^{\bar{i}}(t) = \hat{\zeta}_i^{\bar{i}}(t) - (1 \otimes I_n)\,z_0^{\bar{i}}(t)$ and group formation-tracking error of follower i in subgroup $\bar{i}$ denotes by $\phi_i^{\bar{i}}(t) = x_i^{\bar{i}}(t) - h_i^{\bar{i}}(t) - \zeta_i^{\bar{i}}(t)$.

From Algorithm 1, one gets

$$\begin{aligned}\dot{\phi}_i^{\bar{i}}(t) = (A + BK_2)\,\phi_i^{\bar{i}}(t) + BK_3e_i^{\bar{i}}(t)\\ + Ah_i^{\bar{i}}(t) - \dot{h}_i^{\bar{i}}(t) + Br_i^{\bar{i}}(t)\end{aligned} \tag{24}$$

Because the formation feasibility condition (22) is satisfying, it can be verified that

$$\lim_{t\to\infty}\left(B_2Ah_i^{\bar{i}}(t) - B_2\dot{h}_i^{\bar{i}}(t) + B_2Br_i^{\bar{i}}(t)\right) = 0 \tag{25}$$

Based on calculate compensation input, one has

$$B_1Ah_i^{\bar{i}}(t) - B_1\dot{h}_i^{\bar{i}}(t) + B_1Br_i^{\bar{i}}(t) = 0 \tag{26}$$

Therefore, form (25) and (26), it can be obtained that

$$Ah_i^{\bar{i}}(t) - \dot{h}_i^{\bar{i}}(t) + Br_i^{\bar{i}}(t) \tag{27}$$

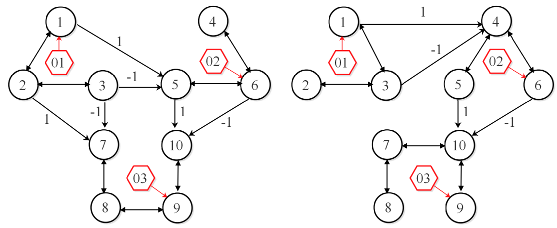
Note that based on Theorem 1 $\lim\limits_{t\to 0}\left(e_i^{\bar{i}}(t)\right) = 0$ and $A + BK_2$ is Hurwitz, thus $\lim\limits_{t\to 0}\left(\phi_i^{\bar{i}}(t)\right) = 0$, which means the group formation tracking error is converge to zeros and the systems are said to realize the group formation tracking under the influence of both time delays and switching topologies. This complete the proof of Theorem 3. ■

Remark 1: According to Theorem 2, only the formation satisfying the formation feasibility condition can be realized. And based on the protocol, the group formation tracking problem in a more complex environment is proven to achieve.

IV. Numerical simulations

In this section, an illustrative simulation is shown to verify the effectiveness of the proposed protocol and algorithm.

Consider a MAS with 13 agents and divided into 3 subgroups. Let $V_1 = \{1, 2, 3\}$, $V_2 = \{4, 5, 6\}$ and $V_3 = \{7, 8, 9, 10\}$ denote the followers of each subgroup. The nodes number of each group are defied as $n_1 = 3$, $n_2 = 3$, $n_3 = 4$. Moreover, three groups of the followers track the trajectory of each leader. Let $\tau(t) = 0.05 + 0.01\cos(t)$. The switching interaction topologies are shown in the Fig. 1



(a) G_1 (b) G_2

Fig. 1: Switching topologies

The system matrixes of the MAS are given as

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ -2 & -6 & -3 \end{bmatrix},\ B = \begin{bmatrix} 0 & -1 \\ -1 & 0 \\ 0 & 0 \end{bmatrix}$$

The desired formations for each group are shown as follow

$$h_i^{\bar{i}}(t) = \begin{bmatrix} r\sin\left(t + \frac{(i-1)2\pi}{n_{\bar{i}}}\right) \\ -r\cos\left(t + \frac{(i-1)2\pi}{n_{\bar{i}}}\right) \\ r\cos\left(t + \frac{(i-1)2\pi}{n_{\bar{i}}}\right) \end{bmatrix}$$

where $i = \{1, 2, \cdots, 10\}$, $\bar{i} = \{1, 2, 3\}$ and $r = 15m$. It can be obtained that the formation tracking feasibility conditions are satisfied. The formation compensation inputs are given as $r_i^{\bar{i}}(t) = 0$.

Based on Algorithm 1, the gain matrixes are given as following

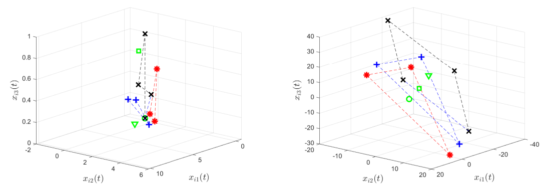
$$K_1 = \begin{bmatrix} 0.4688 & 0.5862 & 0.4935 \\ 0.0096 & 0.6443 & 0.0808 \\ 0.0878 & -0.7426 & -0.0889 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 0.1 & 2.3 & 0.7 \\ -2.7 & -0.1 & -0.9 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} -0.1 & -2.3 & -0.7 \\ 2.7 & 0.1 & 0.9 \end{bmatrix}.$$

The initial states of the leaders and followers are chosen as $x_i^{\bar{i}}(0) = 2(\Theta - 0.5)\,(i = \{1, 2, \cdots, 10\}; \bar{i} = \{1, 2, 3\})$ and $z_0^{\bar{i}}(0) = 2(\Theta - 0.5)\,(\bar{i} = \{1, 2, 3\})$, where Θ is a pseudorandom value that satisfies the uniform distribution between $(0, 1)$. The initial value of the observes are zero.

Fig. 2 shows the state snapshot of 13 agents with $t = 0s$ and $48s$, respectively. Each group formation is denoted by different color. Fig. 3 denotes that the state observers' error can converge to zero within $t = 50s$, which means the observer for each follower can estimate the state of the leader's state in its subgroup. In Fig. 4, group formation tracking error is also convergent, therefore, the three groups



(a) $t = 0s$ (b) $t = 48s$

Fig. 2: Snapshots of seven agents ($t = 0s; t = 48s$)

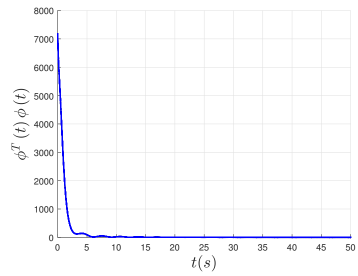


Fig. 3: group formation tracking error within $t = 50s$

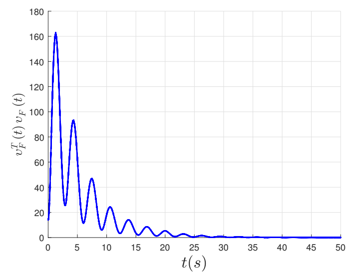


Fig. 4: state observers' error $t = 50s$

of the MASs can achieve the group formation-tracking at last.

V. Conclusions

Time-varying group formation tracking problems under the influence of both communication delays and switching topologies are investigated in this paper. To solve the multiple constraint conditions, a distributed observer is proposed for each subgroup follower to evaluate the state of leader in the subgroup and the designing approach is also put forward based on LMI technique. Then, the effectiveness of the ability to estimate is demonstrated and an observer-based protocol is given for each follower. Moreover, the convergence of the group-formation tracking based on the proposed controller is also presented, which means the systems can realize the group-formation tracking problems with both varying time-delays and switching network.

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