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Output group formation tracking control for heterogeneous systems with collision avoidance and connectivity maintenance

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Abstract: This paper investigates the output group formation tracking problem for heterogeneous systems with collision avoidance and connectivity maintenance methods, and the group formation tracking error can be controlled within an arbitrarily small bound. Firstly, the distributed observers are proposed to estimate the leader's state for each group and the state of itself, respectively. Through constructing the desired potential fields and deviating to get the negative forces, the high safety control protocol is put forward. Moreover, the algorithm consisting of several equations and inequalities is given to calculate the parameters in the controller. By using Lyapunov stability theory, the stability of the error' systems have been analyzed. At last, the effectiveness of the proposed method has been verified by an experiment about the atmospheric monitoring task.

Key Words: Time-varying group formation tracking, Heterogeneous multi-agent system, Collision avoidance and connectivity maintenance

1 Introduction

In recent decades, due to the benefits of high robustness and low consumption, cooperative control has been applied in diversified fields, for instance, underwater vehicles [1, 2], mobile robot systems [3, 4], and missile systems [5, 6].

As the mainstream of consensus control, formation control enlightened by the natural behavior has been a hot issue, where all agents in the systems are required to realize the desired shape. In [7], Ren firstly presented a consensus-based formation strategy, in which the general framework of consensus-building was also put forward. Inspiring by this pioneering work, Dong showed the detailed analysis and design about time-varying formation in [8]. In the meantime, the proposed algorithm had been verified through the unmanned aerial vehicle(UAV). Nevertheless, in practical applications, the classification of the agents will make a great difference, i.e., the agents can be divided into two categories, leaders and followers, respectively. Instead of just maintaining a barren formation, the followers can not only form the desired shape, but also track the trajectory of the leader. Thus, the formation tracking problem comes into being. By dividing the set of followers into two categories, well-informed and uninformed ones, time-varying formation tracking with multiple leaders was investigated in [9]. In practical environments, agents inevitably bump with each other, which implies the requirement of collision avoidance and connectivity maintenance comes into the stage. Based on two novel potential functions, the modified scheme could ensure the collision avoidance and connectivity preservation of multiple mobile agents in [10]. Moreover, Shi et al. [11] proposed a formation tracking protocol with a collision avoidance method for second-order nonlinear systems.

However, considering more complex missions, heterogeneous swarm systems are more powerful than homogeneous ones. For instance, in atmospheric monitoring task, due to the ability of easy operation and high stability, unmanned ground vehicles(UGVs) can deal with emergencies and monitor the lower atmosphere. In the counterpart, UAVs accompanying a wide range of vision can provide successive monitoring at high altitudes. In order to make up for the inconsistency of the dynamic, cooperative output regulators were put forward to solve the heterogeneous systems in [12–14]. Based on the aforementioned strategies, both analyses and experiments were conducted on formation tracking problems for heterogeneous swarm systems in [15]. Furthermore, when it comes to collision avoidance constraints, a collision-free formation tracking problem of heterogeneous robots was proposed in [16], where formation tracking error could be convergent within a given finite time. The attention should be paid that the above works only focus on small-scale multi-robot systems, which means that the whole systems are only related to a single group. there exist different altitudes that should be monitored, group control is more meaningful. Through restricting the order of systems, group consensus of first-order and second-order systems had investigated in [17] and [18], respectively. Inspiring by the above technologies, distributed time-varying group formation tracking had been studied in [19]. Han et al. [20] investigated group formation tracking problems with communication delays. Based on three-layer architecture, group formation tracking control for heterogeneous multi-agent systems was studied in [21].

Note that most of the above-mentioned studies on group formation tracking only focus on communication constraints. In practice, when the scale of individuals was enlarged, the urgent about guaranteeing the safe distance of the systems comes into being, which implies that the

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high safety group formation tracking problem with collision avoidance and connectivity maintenance method for heterogeneous systems needs further consideration.

Motivated by the above analysis, this paper aims to investigate the high-safety group formation tracking control for heterogeneous systems, which can guarantee the distances of individuals into a suitable range. The contributions of this work are summarized as follows

- The heterogeneity of the systems is taken into consideration. Compared with the work in [19] and [20], different orders are taken into account. Since homogeneous systems can be regarded as a special case of heterogeneous systems, the proposed algorithm is more widely used.
- As the combination of the potential function and heterogeneous group formation tracking control, a high safety control protocol with collision avoidance and connectivity maintenance methods is investigated in this paper. Different from normal heterogeneous group formation tracking problems in [21], each robot in this study can maintain the desired distance to avoid collision avoidance and and keep communication. Note that the collision-free methods in [10, 11, 16] can not deal with large-scale systems.

The remainder of this paper is given as follows, Section 2 illustrates the mathematical preliminaries and statements of this work. Detailed analysis is given in Section 3, where the algorithm and proof are demonstrated. The numerical simulation is provided in Section 4 and the conclusion is shown in Section 5.

Notations: Throughout the whole paper, $\mathbb{R}^n$ represents the real vector with $n \times 1$ dimension. The Kronecker product is defined as $\otimes$. $diag\{d_1, d_2, \cdots, d_{N+M}\}$ means diagonal block matrix with d_i as its' diagonal entry.

2 Basic theory and problem statements

2.1 Graph theory

Considering a heterogeneous swarm system consisting of M followers and N leaders. The whole system can be divided into N groups, and each group contains one leader and g_i followers, which means $\sum_{i=1}^{N} g_i = M$. Let $G = \{\mathscr{W}, \mathscr{E}, \mathscr{A}\}$ describe the interaction among the system, where $\mathscr{W} = \{w_1, w_2, \cdots, w_{N+M}\}$, $\mathscr{E} \subseteq \mathscr{W} \times \mathscr{W}$, and $\mathscr{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$ denote the set of nodes, edges, and adjacent matrix, respectively. An undirected graph is defined as connected if there exists a path between each pair of vertices. The degree of G is defined as $\mathscr{D} = diag\{d_1, d_2, \cdots, d_{N+M}\}$ with $d_i = \sum_{j=1}^{N+M} a_{ij}$. The Laplacian matrix of G is given as L, where $L = \mathscr{D} - \mathscr{A}$. The corresponding Laplacian matrix can be given as follows

$$L = \begin{bmatrix} L_F & L_{FE} \\ 0 & 0 \end{bmatrix}$$

where $L_{FE} \in \mathbb{R}^{M\times N}$ is the Laplacian matrix among the followers and leaders. The Laplacian matrix among the followers is denoted as $L_F \in \mathbb{R}^{M\times M}$ which has the following form

$$L_F = \begin{bmatrix} L_{\bar{1}} & 0 & \cdots & 0 \\ L_{\bar{2}\bar{1}} & \ddots & & 0 \\ \vdots & L_{\bar{i}\bar{j}} & \ddots & \vdots \\ L_{\bar{N}\bar{1}} & \cdots & \cdots & L_{\bar{N}} \end{bmatrix}$$

where $L_{\bar{i}}$ represents the Laplacian matrix of the group $\bar{i}$. The interaction Laplacian matrix between the group $\bar{i}$ and $\bar{j}$ is denoted as $L_{\bar{i}\bar{j}}$. $L_{\bar{i}}$ is assumed to be connected.

2.2 Problem statement

The dynamic of the follower i $(i \in \{1, 2, \cdots, M\})$ can be modeled as

$$\begin{cases} \dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) \\ y_i(t) = C_i x_i(t) \end{cases} \tag{1}$$

where $x_i(t) \in \mathbb{R}^{n_i}$, $u_i(t) \in \mathbb{R}^{m_i}$, $y_i(t) \in \mathbb{R}^p$ are denoted as the state, input, and output of i-th follower , respectively. $A_i \in \mathbb{R}^{n_i\times n_i}$, $B_i \in \mathbb{R}^{n_i\times m_i}$, and $C_i \in \mathbb{R}^{p\times n_i}$ are the systems matrices. Supposed that rank$(B_i) = m_i$, (A_i, B_i) is stabilizable, and (C_i, A_i) is detectable.

For group $\bar{i}$ $(\bar{i} \in \{1, 2, \cdots, N\})$, the dynamic of the leader is given as

$$\begin{cases} \dot{x}_{0\bar{i}}(t) = A_0 x_{0\bar{i}}(t) \\ y_{0\bar{i}}(t) = C_0 x_{0\bar{i}}(t) \end{cases} \tag{2}$$

where $x_{0\bar{i}}(t) \in \mathbb{R}^{n_{0\bar{i}}}$ and $y_{0\bar{i}}(t) \in \mathbb{R}^{p_{0\bar{i}}}$ are the state and output of $\bar{i}$-th leader. $A_0 \in \mathbb{R}^{n_0\times n_0}$ and $C_0 \in \mathbb{R}^{p_0\times n_0}$ are denoted as the system matrices.

The predefined formation vector can be defined as $h_{ix}(t) \in \mathbb{R}^{n_{0\bar{i}}}$. Then, the corresponding output formation vector is denoted as $h_{yi}(t)$ satisfying $h_{yi}(t) = C_0 h_{ix}(t)$.

Definition 1. *For any bouned initial states of i-th follower in $\bar{i}$-th group, the following equation is satisfied*

$$\lim_{t\to\infty} (y_i(t) - h_{yi}(t) - y_{0\bar{i}}(t)) = 0 \tag{3}$$

heterogeneous systems (1) and (2) are said to achieve output group formation tracking control.

Assumption 1. *For each follower, the following regulator have a pair solution (X_i, U_i) $(i = 1, 2, \cdots, N)$*

$$\begin{aligned} X_i A_0 &= A_i X_i + B_i U_i \\ 0 &= C_i X_i - C_0 \end{aligned} \tag{4}$$

Assumption 2. *Assuming that there exist no potential fields between the group of leaders.*

Remark 1. *Since each group will execute tasks at different heights and locations, the distances between the leaders are within the desired range, which means the collision avoidance and connectivity maintenance mechanism is no need to be considered between the leaders.*

3 Main results

In the following, the algorithm and analysis about the high safety group formation tracking problems are put forward. Since the process with collision avoidance and connectivity maintenance is instant, the protocol without and with safety precautions methods are both presented. Moreover, the stabilities of the closed-loop systems are analyzed.

3.1 Heterogeneous group formation tracking without safety precaution methods

Consider the following group formation tracking protocol without collision avoidance and connectivity maintenance methods for i-th follower in $\bar{i}$-th group

$$u_i(t) = K_{1i}\hat{x}_i(t) + K_{2i}(\hat{\eta}_i(t) + h_{ix}(t)) + r_i(t) \tag{5}$$

where,

$$\dot{\hat{x}}_i(t) = A_i\hat{x}_i(t) + B_i u_i(t) + L_{0i}(C_i\hat{x}_i(t) - y_i(t)),$$

$$\dot{\hat{\eta}}_i(t) = A_0\hat{\eta}_i(t) - \mu K_0\Bigg[\sum_{j\in N_i, j\in\bar{i}} w_{ij}(\hat{y}_i(t) - \hat{y}_j(t)) + \sum_{j\in N_i, j\notin\bar{i}} \omega_{ij}(\hat{y}_i(t) - \hat{y}_j(t)) + \omega_{i0\bar{i}}(\hat{y}_i(t) - y_{0\bar{i}}(t))\Bigg].$$

$\hat{x}_i(t)$ denotes the state observer of itself. The distributed observer for the leader in the corresponding group is given as $\hat{\eta}_i(t)$. $\hat{y}_i(t)$ denotes the output of the observer satisfying $\hat{y}_i(t) = C_0\hat{\eta}_i(t)$. K_{1i}, K_{2i}, L_{0i}, and K_0 are the gain feedback matrices to be determined later. The formation compensation item is defined as $r_i(t)$. μ is the constant parameter.

Since rank$(B_i) = m_i$, there exist the nonsingular matrix $T_i = \left[\bar{B}_i^T, \hat{B}_i^T\right]^T$ satisfying $\bar{B}_i B_i = I_{m_i}$, $\hat{B}_i B_i = 0$, $\bar{B}_i \in \mathbb{R}^{m_i\times n_i}$, and $\hat{B}_i \in \mathbb{R}^{(n_i-m_i)\times n_i}$.

Algorithm 1: The algorithm for the heterogeneous swarm systems (1) and (2) is given as

Step 1: Calculating (X_i, U_i) through the regulator equations (4).

Step 2: For a given formation vector $h_{ix}(t)$, calculate the following formation feasibility condition

$$\hat{B}_i X_i\left(A_0 h_{ix}(t) - \dot{h}_{ix}(t)\right) = 0 \tag{6}$$

If the above equation satisfies for all followers, the algorithm proceeds sequentially; otherwise, the formation vector should be redesigned.

Step 3: The formation compensation $r_i(t)$ can be put forward as

$$r_i(t) = -\bar{B}_i X_i\left(A_0 h_{ix}(t) - \dot{h}_{ix}(t)\right) \tag{7}$$

Step 4: Defining the proper μ according to $\mu \le \frac{1}{2\mathrm{Re}(\lambda_{Fi,\min})}$. Calculating the positive definite matrix P_0 through the following equation

$$A_0P_0 + P_0A_0^T - P_0C_0^TC_0P_0 + Q = 0 \tag{8}$$

where Q denotes the predefined positive definite matrix. Furthermore, K_0 is given as $K_0 = P_0C_0^T$.

Step 5: Designing K_{1i}, L_{0i}, and K_{2i} through the requirements that $A_i + B_iK_{1i}$ and $A_i + L_{0i}C_i$ are Hurwitz, and $K_{2i} = U_i - K_{1i}X_i$.

Theorem 1. *Under the parameters calculated by Algorithm 1, the leader's observer and state observer can estimate the state of the leader for each group and the state of itself, respectively.*

Proof. Define $\hat{\eta}(t) = \left[\hat{\eta}_1(t)^T, \hat{\eta}_2(t)^T, \cdots, \hat{\eta}_N(t)^T\right]^T$ and $\bar{x}_0(t) = \Bigg[\underbrace{x_{01}^T, \cdots, x_{01}^T}_{g_1}, \cdots, \underbrace{x_{0N}^T, \cdots, x_{0N}^T}_{g_N}\Bigg]^T$. Since $\hat{y}_i(t) = C_0\hat{\eta}_i(t)$ and $y_{0\bar{i}}(t) = C_0x_{0\bar{i}}(t)$, the state observer of the leader can be written into the following compact form

$$\begin{aligned}\dot{\hat{\eta}}(t) = &(I_N \otimes A_0)\hat{\eta}(t) \\ &- \mu(L_F \otimes KC_0)(\hat{\eta}(t) - \bar{x}_0(t))\end{aligned} \tag{9}$$

Let $\tilde{\eta}_i(t) = \hat{\eta}_i(t) - x_{0\bar{i}}(t)$, $\tilde{\eta} = \left[\tilde{\eta}_1^T, \tilde{\eta}_2^T, \cdots, \tilde{\eta}_N^T\right]^T$ be the error of the leader's state observer. Based on (9), one has

$$\dot{\tilde{\eta}}(t) = (I_N \otimes A_0 - \mu(L_F \otimes KC_0))\tilde{\eta}(t) \tag{10}$$

Define λ_{Fi} ($i = 1, 2, \cdots, M$) be the eigenvalues of L_F, and $\lambda_F = diag(\lambda_{F1}, \lambda_{F2}, \cdots, \lambda_{FM})$. Based on the connected assumption for each group, there exist the nonsingular matrix U_F satisfying $U_F^{-1}L_FU_F = \lambda_F$. Let $\tilde{\eta}_F(t) = \left(U_F^{-1} \otimes I_{n_0}\right)\tilde{\eta}(t)$, one has

$$\dot{\tilde{\eta}}_F(t) = (I_N \otimes A_0 - \mu(\lambda_F \otimes KC_0))\tilde{\eta}_F(t) \tag{11}$$

Consider the following Lyapunov candidate function

$$V_{\tilde{\eta}}(t) = \tilde{\eta}_F^H(t)\left(I_N \otimes P_0^{-1}\right)\tilde{\eta}_F(t) \tag{12}$$

Because $K_0 = P_0C_0^T$, the above function can be derived with respect to time

$$\begin{aligned}\dot{V}_{\tilde{\eta}_F}(t) = \tilde{\eta}_F^H(t)\big(&I_N \otimes \left(P_0^{-1}A_0 + A_0^TP_0^{-1}\right) \\ &-2\mu\lambda_F \otimes C_0^TC_0\big)\tilde{\eta}_F(t)\end{aligned} \tag{13}$$

Let $\bar{\eta}_{Fi}(t) = P_0^{-1}\tilde{\eta}_{Fi}(t)$. Since $\mu \le \frac{1}{2\mathrm{Re}(\lambda_{Fi,\min})}$, one has $\dot{V}_{\tilde{\eta}_F}(t) \le -\sum_{i=1}^{N}\bar{\eta}_{Fi}^H(t)Q\bar{\eta}_{Fi}(t)$. It can be concluded that the error of the leader's observer is convergent, which means $\lim_{t\to\infty}\left(\hat{\eta}_i(t) - x_{0\bar{i}}(t)\right) = 0$.

The later part will prove the convergence of the state observer. Define $e_{xi}(t) = \hat{x}_i(t) - x_i(t)$, one has $\dot{e}_{xi}(t) = (A_i + L_{0i}C_i)e_{xi}(t)$. Since L_{0i} suiting the requirement that $A_i + L_{0i}C_i$ is Hurwitz, the observer can estimate the state of itself, which implies $\lim_{t\to\infty}(\hat{x}_i(t) - x_i(t)) = 0$. This completes the proof. □

Theorem 2. *Based on the leader's observer, the state observer of itself, and the gain matrices calculated by Algorithm 1, heterogeneous swarm systems (1) and (2) can achieve the desired output group formation tracking control.*

Proof. Based on the control protocol (5), equation (1) can be written as

$$\begin{aligned}\dot{x}_i(t) = A_ix_i(t) + B_i(&K_{1i}\hat{x}_i(t) + \\ &K_{2i}(\hat{\eta}_i(t) + h_{ix}(t)) + r_i(t))\end{aligned} \tag{14}$$

Define $H_i(t) = x_i(t) - X_i(x_{0\bar{i}}(t) + h_{ix}(t))$. Based on (14) and (2), one gets

$$\begin{aligned}\dot{H}_i(t) = &(A_i + B_iK_{1i})H_i(t) \\ &+B_iK_{1i}e_{xi}(t) + B_iK_{2i}\tilde{\eta}_i(t) \\ &+X_iA_0h_{ix}(t) - X_i\dot{h}_{ix}(t) + B_ir_i(t)\end{aligned} \tag{15}$$

where $e_{xi}(t) = \hat{x}_i(t) - x_i(t)$, $\tilde{\eta}_i(t) = \hat{\eta}_i(t) - x_{0\bar{i}}(t)$. According to the formation feasibility condition and formation compensation condition, one has

$$\lim_{t\to\infty}\Big(X_i A_0 h_{ix}(t) - X_i \dot{h}_{ix}(t) + B_i r_i(t)\Big) = 0 \tag{16}$$

Since K_{1i} satisfying $A_i + B_i K_{1i}$ is Hurwitz, $\lim_{t\to\infty} e_{xi}(t) = 0$, $\lim_{t\to\infty} \tilde{\eta}_i(t) = 0$, moreover, based on the Input-state stability theory, one has $\lim_{t\to\infty} H_i(t) = 0$. Note that $H_{yi}(t) = C_i H_i(t) = y_i(t) - h_{yi}(t) - y_{0\bar{i}}(t)$, one gives $\lim_{t\to\infty} H_{yi}(t) = 0$. Then, it can be proven that the heterogeneous systems (1) and (2) can realize output group formation tracking control. □

3.2 Heterogeneous group formation tracking with collision avoidance and connectivity maintenance

In many practical environments, the individuals of swarm systems can bump into each other due to the inappropriate initial state. In the meantime, the whole system should maintain the desired communication distance. Thus, it is urgent to investigate the collision avoidance and connectivity maintenance methods in group formation tracking problem.

Due to the abilities of high interpretability and easy operation, the artificial potential fields are put forward to incorporate the controller to maintain the communication distance and safety distance. The repulsive force and attractive force are defined as follow

$$F_{CA,ij}(D_{ij}(t)) = \begin{cases} if\ D_{ij}(t) \in (D_{CA,\min}, D_{CA,\max}] \\ \frac{k_{CA} D_{ij}(t)}{\left\| D_{ij}(t) - D_{CA,\min} \right\|^{v_{CA}}} \frac{(y_i(t) - y_j(t))}{D_{ij}(t)} \\ if\ D_{ij}(t) \in (D_{CA,\max}, D_{CA,\max,\exp}] \\ f_{CA}(D_{ij}(t)) \frac{(y_i(t) - y_j(t))}{D_{ij}(t)} \\ if\ D_{ij}(t) \in (D_{CA,\max,\exp}, +\infty] \\ 0 \end{cases} \tag{17}$$

$$F_{CM,ij}(D_{ij}(t)) = \begin{cases} if\ D_{ij}(t) \in (D_{CM,\min}, D_{CM,\max}] \\ \frac{-k_{CM} D_{ij}(t)}{\left\| D_{ij}(t) - D_{CM,\max} \right\|^{v_{CM}}} \frac{(y_i(t) - y_j(t))}{D_{ij}(t)} \\ if\ D_{ij}(t) \in (D_{CM,\min,\exp}, D_{CM,\min}] \\ f_{CM}(D_{ij}(t)) \frac{(y_i(t) - y_j(t))}{D_{ij}(t)} \\ if\ D_{ij}(t) \in (0, D_{CM,\min,\exp}] \\ 0 \end{cases} \tag{18}$$

where $D_{ij}(t) = \|y_i(t) - y_j(t)\|$, $D_{CA,\max,\exp} > D_{CA,\max} > D_{CA,\min} > 0$ and $D_{CM,\max} > D_{CM,\min} > D_{CM,\min,\exp} > 0$ are the predefined safe distance. k_{CA}, v_{CA}, k_{CM} and v_{CM} are the parameters of functions. In order to prevent the control input from jumping, $f_{CA}(D_{ij}(t))$ and $f_{CM}(D_{ij}(t))$ are designed as follow

$$\begin{cases} f_{CA} = \bar{f}_{CA}\cos\left[\omega_{CA}(D_{ij}(t) - D_{CA,\max})\right] + \bar{f}_{CA} \\ f_{CM} = -\bar{f}_{CM}\cos\left[\omega_{CM}(D_{ij}(t) - D_{CA,\min,\exp})\right] \\ \qquad + \bar{f}_{CM} \end{cases} \tag{19}$$

where

$$\bar{f}_{CA} = \frac{k_{CA} D_{CA,\max}}{2\|D_{CA,\max} - D_{CM,\min}\|^{v_{CA}}},$$
$$\omega_{CA} = \frac{\pi}{D_{CA,\max,\exp} - D_{CA,\max}},$$
$$\bar{f}_{CM} = \frac{-k_{CM} D_{CM,\min}}{2\|D_{CM,\min} - D_{CM,\max}\|^{v_{CM}}},$$
$$\omega_{CM} = \frac{\pi}{D_{CM,\min} - D_{CM,\min,\exp}}.$$

Through integrating, the potential functions can be derived as follow

$$\begin{cases} V_{CA,ij} = \int_0^{D_{ij}(t)} (-F_{CA,ij}(D_{ij}(t))) d(D_{ij}(t)) \\ V_{CM,ij} = \int_0^{D_{ij}(t)} (-F_{CM,ij}(D_{ij}(t))) d(D_{ij}(t)) \end{cases} \tag{20}$$

Then, the control protocol is shown as

$$\begin{aligned} u_i(t) = {} & K_{1i}\hat{x}_i(t) + K_{2i}(\hat{\eta}_i(t) + h_{ix}(t)) \\ & + r_i(t) - K_{3i} C_i^T F_i(t) \end{aligned} \tag{21}$$

where

$$\dot{\hat{x}}_i(t) = A_i\hat{x}_i(t) + B_i u_i(t) + L_{0i}(C_i\hat{x}_i(t) - y_i(t)),$$
$$\dot{\hat{\eta}}_i(t) = A_0\hat{\eta}_i(t) - \mu K_0 \Bigg[\sum_{j\in N_i, j\in\bar{i}} w_{ij}\left(\hat{y}_i(t) - \hat{y}_j(t)\right) +$$
$$\sum_{j\in N_i, j\notin\bar{i}} \omega_{ij}\left(\hat{y}_i(t) - \hat{y}_j(t)\right) + \omega_{i0\bar{i}}\left(\hat{y}_i(t) - y_{0\bar{i}}(t)\right)\Bigg],$$
$$F_i(t) = \sum_{j=1}^{N} (F_{CA,ij}(t) + F_{CM,ij}(t)).$$

Apart from the same parameters in the protocol (5), K_{3i} is the gain feedback matrix to be determined later. $F_i(t)$ denotes the sum of forces of i-th follower.

Theorem 3. *If K_{3i} meets $K_{3i} = -\bar{B}_i M_i^{-1}(A_i + B_i K_{1i})^T$, where $M_i \in \mathbb{R}^{n_i\times n_i}$ satisfying the following Lyapunov function*

$$(A_i + B_i K_{1i})^T M_i + M_i(A_i + B_i K_{1i}) = -2I_{n_i} \tag{22}$$

heterogeneous systems (1) and (2) are said to realize the output group formation tracking and achieve collision avoidance and connectivity maintenance mechanism.

Proof. By incorporating the protocol into (1), one gets

$$\begin{aligned} \dot{x}_i(t) = {} & A_i x_i(t) + B_i\big(K_{1i}\hat{x}_i(t) + K_{2i}(\hat{\eta}_i(t) \\ & + h_{ix}(t)) + r_i(t) - K_{3i} C_i^T F_i(t)\big) \end{aligned} \tag{23}$$

Let $H_i(t) = x_i(t) - X_i(x_{0\bar{i}}(t) + h_{ix}(t))$. According to (23) and (2), it can be concluded that

$$\begin{aligned} \dot{H}_i(t) = {} & (A_i + B_i K_{1i}) H_i(t) - B_i K_{3i} C_i^T F_i(t) \\ & + B_i K_{1i} e_{xi}(t) + B_i K_{2i}\tilde{\eta}_i(t) + \varphi_i(t) \end{aligned} \tag{24}$$

where

$$e_{xi}(t) = \hat{x}_i(t) - x_i(t),$$
$$\tilde{\eta}_i(t) = \hat{\eta}_i(t) - x_{0\bar{i}}(t),$$
$$\varphi_i(t) = X_i A_0 h_{ix}(t) - X_i\dot{h}_{ix}(t) + B_i r_i(t).$$

Based on formation feasibility condition and formation compensation parameter, one has $\lim_{t\to\infty}\varphi_i(t) = 0$.

Considering the following Lyapunov candidate function

$$V_{H_i}(t) = V_{H_{i1}}(t) + V_{H_{i2}}(t) \tag{25}$$

where $V_{H_{i1}} = H_i^T(t) M_i H_i(t)$ and $V_{H_{i2}} = 2V_i(t)$. $V_i(t)$ satisfying $V_i(t) = \sum_{j=1}^{N}(V_{CA,ij}(D_{ij}(t)) + V_{CM,ij}(D_{ij}(t)))$.

Taking the derivative of the above Lyapunov function with respect to time, one has

$$\dot{V}_{H_i}(t) = \dot{V}_{H_{i1}}(t) + \dot{V}_{H_{i2}}(t) \tag{26}$$

where,

$$\begin{aligned}&\dot V_{H_{i1}}(t) = H_i^T(t)\left[(A_i+B_iK_{1i})^T M_i\right.\\&\left.+M_i(A_i+B_iK_{ii})\right]H_i(t)+2H_i^TM_iB_iK_{1i}e_{xi}(t)\\&+2H_i^TM_iB_iK_{2i}\tilde\eta_i(t)+2H_i^TM_i\varphi_i(t)\\&-2H_i^TM_iB_iK_{3i}C_i^TF_i(t),\end{aligned}$$

$$\begin{aligned}&\dot V_{H_{i2}}(t) = -2H_i^T(t)(A_i+B_iK_{1i})^TC_i^TF_i(t)\\&+2F_i^T(t)C_iB_iK_{3i}C_i^TF_i(t)-2F_i^T(t)C_iB_iK_{1i}e_{xi}(t)\\&-2F_i^T(t)C_iB_iK_{2i}\tilde\eta_i(t)-2F_i^T(t)C_i\varphi_i(t)\\&-2F_i^T(t)C_iX_i\dot h_{ix}(t)\end{aligned}$$

Based on Young's inequality, one gets

$$\left\{\begin{array}{l}2H_i^T(t)M_iB_iK_{1i}e_{xi}(t)\le\frac{1}{2}H_i^T(t)H_i(t)+2\|M_iB_iK_{1i}e_{xi}(t)\|^2\\2H_i^T(t)M_iB_iK_{2i}\tilde\eta_i(t)\le\frac{1}{2}H_i^T(t)H_i(t)+2\|M_iB_iK_{2i}\tilde\eta_i(t)\|^2\\2H_i^T(t)M_i\varphi_i(t)\le\frac{1}{2}H_i^T(t)H_i(t)+2\|M_i\varphi_i(t)\|^2\end{array}\right. \tag{27}$$

$$\left\{\begin{array}{l}-2F_i^T(t)C_iB_iK_{1i}e_{xi}(t)\le\delta_1F_i^T(t)F_i(t)+\frac{1}{\delta_1}\|C_iB_iK_{1i}e_{xi}(t)\|^2\\-2F_i^T(t)B_iB_ik_{2i}\tilde\eta_i(t)\le\delta_2F_i^T(t)F_i(t)+\frac{1}{\delta_2}\|C_iB_{ik2i}\tilde\eta_i(t)\|^2\\-2F_i^T(t)C_i\varphi_i(t)\le\delta_3F_i^T(t)F_i(t)+\frac{1}{\delta_3}\|C_i\varphi_i(t)\|^2\\-2F_i^T(t)C_iX_i\dot h_{ix}(t)\le\delta_4F_i^T(t)F_i(t)+\frac{1}{\delta_4}\left\|C_iX_i\dot h_{ix}(t)\right\|^2\end{array}\right. \tag{28}$$

where δ_1, δ_2, δ_3, and δ_4 are the small positive parameters.

Define $2C_iB_iK_{3i}C_i^T = Q_i$. Moreover, based on (27), (28), and $M_iB_iK_{3i} = -(A_i+B_iK_{1i})^T$, it can be seen that

$$\begin{aligned}&\dot V_{H_i}(t)\le-\frac{1}{2\lambda_{\max}(M_i)}V_{H_{i1}}(t)+2\|M_iB_iK_{1i}e_{xi}(t)\|^2\\&+\frac{1}{\delta_1}\|C_iB_iK_{1i}e_{xi}(t)\|^2+2\|M_iB_iK_{2i}\tilde\eta_i(t)\|^2\\&+\frac{1}{\delta_2}\|C_iB_{ik2i}\tilde\eta_i(t)\|^2+2\|M_i\varphi_i(t)\|^2+\frac{1}{\delta_3}\|C_i\varphi_i(t)\|^2\\&+\frac{1}{\delta_4}\left\|C_iX_i\dot h_{ix}(t)\right\|^2+\delta F_i^T(t)F_i(t)\end{aligned} \tag{29}$$

where $\delta = (\lambda_{\max}(Q_i)+\delta_1+\delta_2+\delta_3+\delta_4)$.

It should be pointed out that the duration of collision avoidance and connectivity maintenance is finite. The end time of the above process is defined as t_F. one sees that during the time period $(0, t_F]$, $e_{xi}(t)$, $\tilde\eta_i(t)$, and $\varphi_i(t)$ are bounded. Moreover, due to the definitions, $F_i(t)$ and $\dot h_{ix}(t)$ are also bounded. Then, it can be concluded that $V_{H_i}(t)$ is bounded and the corresponding $H_i(t)$ is bounded.

When $t > t_F$, individuals are not subjected to the forces, namely $F_i(t) = V_i(t) = 0$ and $V_{H_i}(t) = V_{H_{i1}}(t)$. Note that $\lim\limits_{t\to\infty} e_{xi}(t) = 0$, $\lim\limits_{t\to\infty}\tilde\eta_i(t) = 0$, and $\lim\limits_{t\to\infty}\varphi_i(t) = 0$, it can be concluded that $\lim\limits_{t\to\infty} V_{H_i}(t) = 0$, which means $\lim\limits_{t\to\infty} H_i(t) = 0$. By the definition, one has $\lim\limits_{t\to\infty} H_{yi}(t) = 0$. Then, the heterogeneous systems (1) and (2) are proven to achieve the output group formation tracking with collision avoidance and connectivity maintenance method. This completes the proof. □

4 Numerical simulation

Suppose that the experiment consists of nine heterogeneous agents. The followers are divided into two subgroups. It should be pointed out that the heterogeneous swarm system is assigned to perform the atmospheric monitoring task through cooperation. The interaction topology among the whole systems is given as Fig. 1.

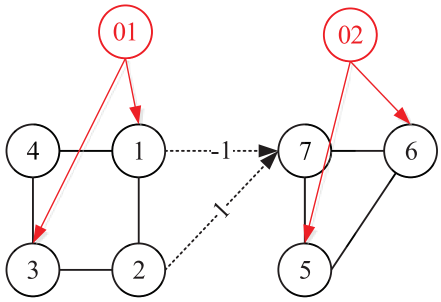


Fig. 1: The interaction topology.

The dynamic of leaders can be given as $A_{01} = A_{02} = I_2\otimes\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}$, $B_{01} = B_{02} = I_2\otimes\begin{bmatrix}0\\1\end{bmatrix}$, and $C_{01} = C_{02} = I_2\otimes\begin{bmatrix}1 & 0\end{bmatrix}$. In addition, the dynamics of followers are shown as $A_1 = A_2 = A_5 = A_6 = I_2\otimes\begin{bmatrix}0 & 1\\-3 & -3\end{bmatrix}$, $A_3 = A_4 = A_7 = I_2\otimes\begin{bmatrix}0 & 1\\-1 & -1.2\end{bmatrix}$, $B_i = I_2\otimes\begin{bmatrix}0\\1\end{bmatrix}$ $(i = 1,2,\cdots,7)$, and $C_i = I_2\otimes\begin{bmatrix}1 & 0\end{bmatrix}$ $(i = 1,2,\cdots,7)$.

Through solving the regulator functions, one gets $X_i = I_2\otimes\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ $(i = 1,2,\cdots,7)$, $U_i = I_2\otimes\begin{bmatrix}2 & 3\end{bmatrix}$ $(i = 1,2,5,6)$, $U_j = I_2\otimes\begin{bmatrix}0 & 1.2\end{bmatrix}$ $(j = 3,4,7)$. Based on the proposed Algorithm, one has $K_{1i} = I_2\otimes\begin{bmatrix}2.5 & 1.5\end{bmatrix}$ $(i = 1,2,5,6)$, $K_{1j} = I_2\otimes\begin{bmatrix}0.5 & -0.3\end{bmatrix}$ $(j = 3,4,7)$, $K_{2i} = I_2\otimes\begin{bmatrix}-0.5 & 1.5\end{bmatrix}$ $(i = 1,2,\cdots,7)$, $L_{0i} = I_2\otimes\begin{bmatrix}0 & 1\end{bmatrix}$ $(i = 1,2,5,6)$, and $L_{0j} = I_2\otimes\begin{bmatrix}-1.8 & 1.16\end{bmatrix}$ $(j = 3,4,7)$. According to the Lyapunov function, M_i is denoted as $M_i = I_2\otimes\begin{bmatrix}4 & 2\\2 & 2\end{bmatrix}$ $(i = 1,2,\cdots,7)$, then $K_{3i} = I_2\otimes\begin{bmatrix}1 & -1.25\end{bmatrix}$ $(i = 1,2,\cdots,7)$.

The predefined formations are given as

$$h_{ix} = \begin{bmatrix}15*\sin\left(t+\frac{(i-1)\pi}{2}\right)\\15*\cos\left(t+\frac{(i-1)\pi}{2}\right)\\15*\cos\left(t+\frac{(i-1)\pi}{2}\right)\\-15*\sin\left(t+\frac{(i-1)\pi}{2}\right)\end{bmatrix}(i = 1,2,3,4)$$

$$h_{jx} = \begin{bmatrix}5*\sin\left(t+\frac{(j-5)2\pi}{3}\right)\\5*\cos\left(t+\frac{(j-5)2\pi}{3}\right)\\5*\cos\left(t+\frac{(j-5)2\pi}{3}\right)\\-5*\sin\left(t+\frac{(j-5)2\pi}{3}\right)\end{bmatrix}(j = 5,6,7)$$

Let $\bar B_i = I_2\otimes\begin{bmatrix}0 & 1\end{bmatrix}$ $(i = 1,2,\cdots,7)$, $\hat B_i = I_2\otimes\begin{bmatrix}1 & 0\end{bmatrix}$ $(i = 1,2,\cdots,7)$, one sees that the formation feasibility condition is satisfied. Moreover, the formation compensation can be given as $r_i = \begin{bmatrix}-15\sin\left(t+\frac{(i-1)\pi}{2}\right)\\-15\sin\left(t+\frac{(i-1)\pi}{2}\right)\end{bmatrix}$ $(i = 1,2,3,4)$, $r_j = \begin{bmatrix}0\\0\end{bmatrix}$ $(j = 5,6,7)$.

The safe distance range of systems is defined as [0.65m, 55m]. Let $D_{CA,\max,\exp} = 1.1\text{m}$, $D_{CA,\max} = 1\text{m}$,

$D_{CA,\min} = 0.65\text{m}$, $k_{CA} = 0.000003$, $v_{CA} = 5$, $D_{CM,\max} = 55\text{m}$, $D_{CM,\min} = 50\text{m}$, $D_{CM,\min,\exp} = 40\text{m}$, $k_{CM} = 0.01$, and $v_{CM} = 3$.

The initial states of the leaders are given as $\begin{bmatrix} 50 & -1 & 1 & 50 \end{bmatrix}^T$ and $\begin{bmatrix} 20 & 1 & -1 & 20 \end{bmatrix}^T$, respectively. Choose zeros as the initial states of the leader's observer and observer of itself. The initial states of the followers are given as $x_i(0) = [50\varpi, \varpi, 50\varpi, \varpi]^T (i = 1, 2, 3, 4)$ and $x_j(0) = [8\varpi, \varpi, 8\varpi, \varpi]^T$ $(j = 5, 6, 7)$. ϖ denotes the random value between -0.5 to 0.5.

The output trajectories of the heterogeneous systems are given as Fig. 2 and Fig. 3.

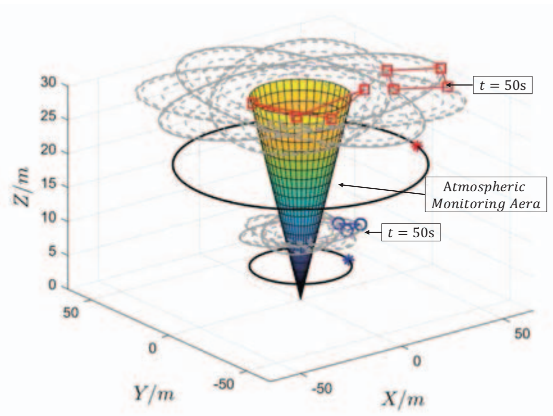


Fig. 2: 3-D trajectories under atmospheric monitoring task.

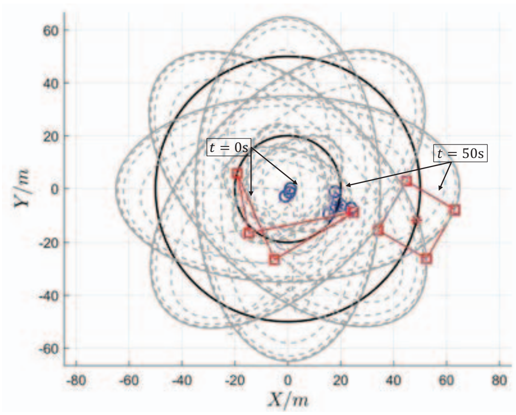
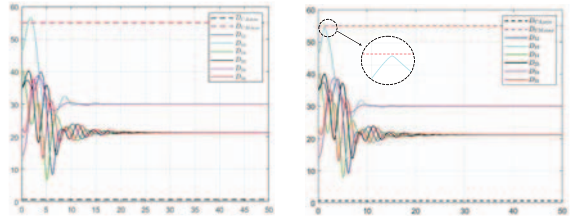


Fig. 3: Plan view.

The individuals of group one and group two are denoted by red and blue, respectively. Let $*$ represent the leader of each group. $\square$ and $\bigcirc$ denote the individuals of group one and two, respectively.

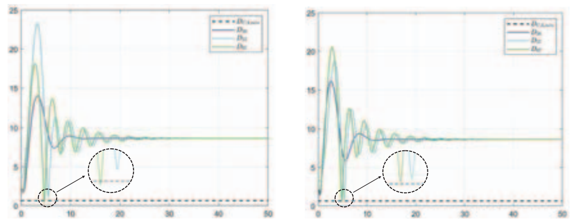
Note that the followers in group one are flying at an altitude of 30m and realizing the desired quadrangular formation to monitor the atmosphere at a high altitude. The leader in group one is flying at an altitude of 20m, rotating with radius of 50m, providing the trajectory of group one, and monitoring the middle and upper atmosphere environment. In the meantime, the followers in group two are flying at an altitude of 10m and realizing the desired triangular formation to monitor the atmosphere at middle and lower altitudes. The leader in group two is flying at an altitude of 5m, rotating with radius of 20m, providing the trajectory of group two, and monitoring the lower atmosphere environment. From the simulation results, the heterogeneous swarm systems can realize the desired output group formation tracking control, and achieve the atmospheric monitoring task at different heights.

In order to prove the effectiveness of the high safety method, the control experiment is conducted, and the results are shown as Fig. 4 and Fig. 5.



(a) *Without method* (b) *With safety method*

Fig. 4: Distances between the individuals of group one.



(a) *Without method* (b) *With safety method*

Fig. 5: Distances between the individuals of group two.

From Fig. 4 and Fig. 5, one sees that if collision avoidance and connectivity maintenance are not taken into consideration, the distances between the individuals of group one will exceed the communication safety distance, and the individuals of group two will collide with each other. In contrast, under the proposed protocol, the distances between each other will maintain the desired range.

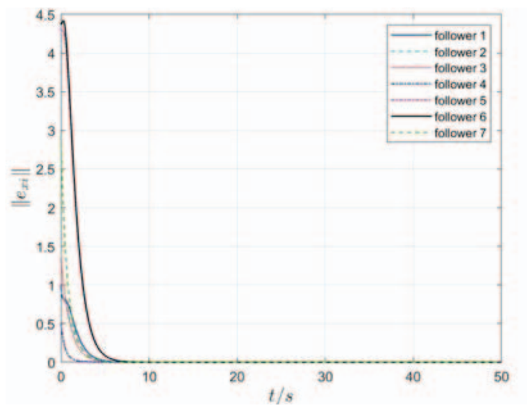


Fig. 6: Errors about the state observer of the group leader.

Fig. 6 and Fig. 7 give state observers' errors of the group leader and the state of itself, respectively. The heterogeneous output group formation tracking errors are given as Fig. 8. one sees that all errors' systems are convergent, which means heterogeneous systems can realize the high safety output group formation tracking.

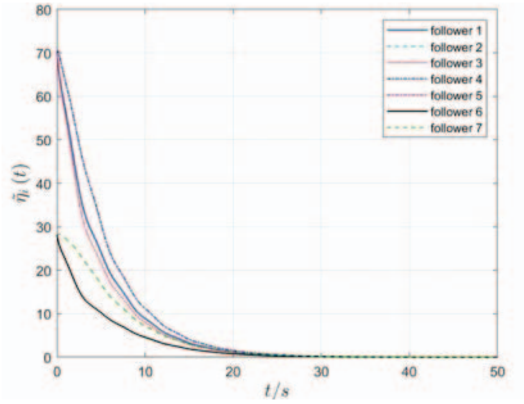


Fig. 7: Errors about the state observer of itself.

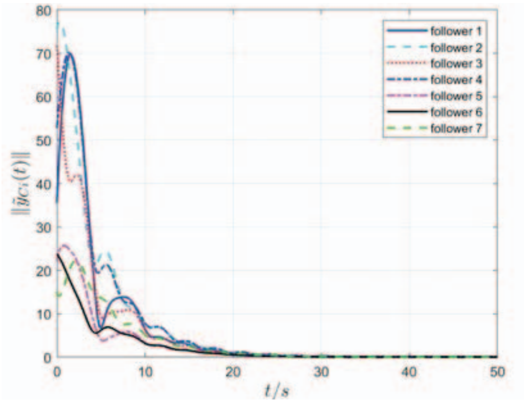


Fig. 8: Output group formation tracking errors.

5 Conclusion

This paper had addressed group formation tracking control of heterogeneous swarm systems with collision avoidance and connectivity maintenance. Firstly, the normal method without safety measures was conducted. Then, based on the predefined artificial potential fields, the controller was put forward to guarantee the safety range, which means if the distance between each agent exceeded the communication safety distance or position safety distance, the proposed forces would play an important role. Moreover, the convergences of the error's systems were also analysed.

References

[1] D. Zhang, Z. Ye, and X. Dong, Co-design of fault detection and consensus control protocol for multi-agent systems under hidden DoS attack. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 68(5): 2158-2170, 2021.

[2] J. Fan, and L. Zhao, Distributed finite-time adaptive consensus tracking control for multiple AUVs with state constraints. *Journal of the Franklin Institute*, 358(17): 9158-9177, 2021.

[3] J. Yu, X. Dong, Q. Li, J. Lv, and Z. Ren, Fully adaptive practical time-varying output formation tracking for high-order nonlinear stochastic multi-agent system with multiple leaders. *IEEE Transactions on Cybernetics*, 51(4): 2265-2277, 2021.

[4] J. Yu, X. Dong, Q. Li, J. Lv, and Z. Ren, Adaptive practical optimal time-varying formation tracking control for disturbed high-order multi-agent systems. *IEEE Transactions on Circuits and Systems I: Regular Papers*, DOI: 10.1109/TCSI.2022.3151464.

[5] J. Yu, X. Dong, Q. Li, and Z. Ren, Cooperative guidance strategy for multiple hypersonic gliding vehicles system. *Chinese Journal of Aeronautics*, 33(3): 990-1005, 2020.

[6] Z. Shen, J. Yu, X. Dong, Y. Hua, and Z. Ren, Penetration trajectory optimization for the hypersonic gliding vehicle encountering two interceptors. *Aerospace Science and Technology*, 121: 107363, 2022.

[7] W. Ren, Consensus based formation control strategies for multi-vehicle systems, in *Proceedings of 25th IEEE American Control Conference*, 2006: 4237–4242.

[8] X. Dong, B. Yu, Z. Shi, and Y. Zhong, Time-varying formation control for unmanned aerial vehicles: Theories and applications. *IEEE Transactions on Control Systems Technology*, 23(1): 340–348, 2014.

[9] X. Dong, and G. Hu, Time-varying formation tracking for linear multiagent systems with multiple leaders. *IEEE Transactions on Automatic Control*, 62(7): 3658–3664, 2017.

[10] A. Mondal, C. Bhowmick, L. Behera, and M. Jamshidi, Trajectory tracking by multiple agents in formation with collision avoidance and connectivity assurance. *IEEE Systems Journal*, 12(3): 2449–2460, 2017.

[11] Q. Shi, T. Li, J. Li, C. P. Chen, Y. Xiao, and Q. Shan, Adaptive leader-following formation control with collision avoidance for a class of second-order nonlinear multi-agent systems. *Neurocomputing*, 350: 282–290, 2019.

[12] P. Wieland, R. Sepulchre, and F. Allgöwer, An internal model principle is necessary and sufficient for linear output synchronization. *Automatica*, 47(5): 1068–1074, 2011.

[13] W. Hu, L. Liu, and G. Feng, Output consensus of heterogeneous linear multi-agent systems by distributed event-triggered/self-triggered strategy. *IEEE Transactions on Cybernetics*, 47(8): 1914–1924, 2017.

[14] Y. Qian, L. Liu, and G. Feng, Output consensus of heterogeneous linear multi-agent systems with adaptive event-triggered control. *IEEE Transactions on Automatic Control*, 64(6): 2606–2613, 2019.

[15] Y. Hua, X. Dong, G. Hu, and Z. Ren, Distributed time-varying output formation tracking for heterogeneous linear multiagent systems with a nonautonomous leader of unknown input. *IEEE Transactions on Automatic Control*, 64(10): 4292–4299, 2019.

[16] K. Wu, J. Hu, B. Lennox, and F. Arvin, Finite-time bearing-only formation tracking of heterogeneous mobile robots with collision avoidance. *IEEE Transactions on Circuits and Systems II: Express Briefs*, 68(10): 3316–3320, 2021.

[17] Y. Feng, and W. X. Zheng, Group consensus control for discrete-time heterogeneous first-and second-order multi-agent systems. *IET Control Theory and Applications*, 12(6): 753–760, 2018.

[18] Y. Feng, S. Xu, and B. Zhang, Group consensus control for double-integrator dynamic multi-agent systems with fixed communication topology. *International Journal of Robust and Nonlinear Control*, 24(3): 532–547, 2014.

[19] J. Hu, P. Bhowmick, and A. Lanzon, Distributed adaptive time-varying group formation tracking for multi-agent systems with multiple leaders on directed graphs. *IEEE Transactions on Control of Network Systems*, 7(1): 140–150, 2019.

[20] L. Han, Y. Xie, X. Li, X. Dong, Q. Li, and Z. Ren, Time-varying group formation tracking control for second-order multi-agent systems with communication delays and multiple leaders. *Journal of the Franklin Institute*, 357(14): 9761-9780, 2020.

[21] L. Tian, Q. Zhao, X. Dong, Q. Li, and Z. Ren, Time varying output group formation tracking for heterogeneous multi-agent systems. *Acta Aeronautica et Astronautica Sinica*, 41(7): 323727, 2020.