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Time-varying group formation-tracking control for heterogeneous multi-agent systems with switching topologies and time-varying delays

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Abstract: Group formation-tracking problem for heterogeneous multi-agent systems (HMASs) with both switching networks and communication delays is investigated in this paper. In order to achieve different tasks, the agents are classified into various groups. The followers are allowed to realize the formation and track the trajectory of the leader in each group. Firstly, by utilizing the consensus control, an observer is proposed to estimate the state of the leader. Then, an observer-based control protocol is put forward to solve the group formation-tracking problem with both communication delays and switching networks. Moreover, an algorithm to determine the gain feedbacks is demonstrated, in the meantime, the observer's error systems as well as the group formation-tracking error systems are proved to be convergent. Finally, an example in the simulation prat is presented to verify the theoretical results.

Key Words: Group formation-tracking problem, general linear multi-agent systems, time-varying delays, switching topologies

1 Introduction

Recently, due to its low computing consumption and high robustness, consensus control has been applied in many fields, for example, unmanned aerial vehicles (UAVs) [1–3], Space vehicles [4–7], etc.

As an important aera in the coordinated control, formation control whose goal is to drive the disorganized agents form the expected formation has aroused considerable interests. As a pioneering work in formation problems, Ren in [8] firstly used consensus strategy for second-order systems to realize the desired formation. Inspired by this intelligent work, Dong in [9] put forward the formation feasibility condition which means not all formation can be achieved, and the theoretical results are applied into the UAVs to verify the results. However, due to the congestion of the interaction channel and complex communication environment, varying communication delays along with switching networks emerge. Xiao in [10] proposed a strategy to solve the generic MASs with both varying delays and switching networks showing that the control protocol can be used in a more practical complex environment. Moreover, in many practical tasks such as coordinated patrolling, etc. The agents are not only allowed to achieve the expected formation but also to track the desired trajectory. Therefore, formation-tracking problem comes into the stage. Considering the switching topologies, a consensus-based control protocol was put forward in the work of Yu et.al [11], and switching time has been given according to the dwelling time. Linear matrices inequality (LMI) along with the consensus protocol is presented to solve the MAS with varying time delays in the work of [12]. It should be mentioned that the aforementioned research only focuses on the single group. In many complicated tasks, the agents should be divided into several groups to realize different missions. Based on the assumption of acyclic partition of the nodes, group formation problems were solved in the work of Qin et al. [13]. Considering the switching topologies, a group formation-tracking protocol and the algorithm to determine the gain matrices were put forward in the work of [14]. Han in [15] investigated the group formation-tracking problem for second order systems with time-varying communication delays.

For the works mentioned above, only homogeneous systems were taken into consideration, which means the agents in the systems are all identical. In order to deal with the HMASs with switching interaction topologies, an unique formation tracking protocol was proposed in [16]. To incorporate continuous repulsive vector into agents' velocity, HMASs with varying communication delay to realize formation-tracking was investigated in [17]. As can be seen from the literatures mentioned above, these works considered the varying delays or switching networks, respectively, but none of them study the output group formation-tracking systems with both varying-time delays and switching networks. Designing a group formation-tracking protocol for HMASs with both communication delays and switching interaction topologies is challenging and still open.

Motivated by the above studies, this paper investigates the output group formation-tracking for HMASs with both varying time delays and switching networks. Compared with the aforementioned researches, the contributions are shown as follows:

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1) As a combination of the output group control and heterogenous systems, the output group formation tracking systems for HMASs are much more realistic. On the one hand, compared with the work in [13–15], different orders are taken into consideration. On the other hand, compared with the work in [10], the robots in this paper can be divided into several groups to execute different missions.
2) Both varying time-delays and switching networks are taken into consideration in this paper, which means compared with the control methods proposed in the works [16, 17], the control protocol are much more versatile and can be used to deal with a more complex communication environment.

The organization of the rest paper is given as follows. Section 2 demonstrates some basic concepts on graph theory and some significant definitions are also proposed. In section 3, a state observer and group formation-tracking protocol are put forward for HMAS with communication delays and switching networks. Moreover, an algorithm is also presented. Section 4 shows a numerical simulation for the mentioned method. Section 5 concludes the whole works.

Throughout this paper, let $\otimes$ denote the Kronecker product of two matrices. I represents an identify matrix with appropriate size. Let $\boldsymbol{1}$ and 0_n be a column vectors with 1 and n as its element.

2 Preliminaries and problem description

In this section, some basic concepts and notations on graph theory and the problem descriptions are demonstrated.

2.1 Preliminaries

A weighted undirected graph G can be represented by $\{V, T, W\}$, where $V = \{v_1, v_2, \cdots v_N\}$ is a set of nodes, $T \subseteq \{(v_i, v_j) : v_i, v_j \in V\}$ is the set of edges, and $W = [a_{ij}] \in \mathbb{R}^{N\times N}$ is a weighted adjacency matrix. Let $e_{ij} = (v_i, v_j)$ denote the edge of G and w_{ij} denote the nonnegative element with e_{ji}. Define $w_{ij} > 0$ if and only if $e_{ji} \in T$ and $w_{ij} = 0$ otherwise. $N_i = \{v_j \in V : (v_j, v_i) \in T\}$ is the set of neighbors of node v_i. The Laplacian matrix L is defined as $L = D - W$, where $D = diag\left\{\sum_{j=1}^{N} w_{1j}, \sum_{j=1}^{N} w_{2j}, \cdots, \sum_{j=1}^{N} w_{Nj}\right\}$. A path from node v_{i1} to v_{ik} is a series of ordered edges (v_{i1}, v_{i2}), (v_{i2}, v_{i3}), $\cdots$, (v_{ik-1}, v_{ik}). The definition of undirected graph is that $v_{ij} \in T$ implies $v_{ji} \in T$ and $w_{ij} = w_{ji}$. The undirected graph is said to be connected if there is a path between any distinct pair of nodes.

It is assumed that the interaction topologies are switching. Let $[t_k, t_{k+1})$ $(k \in \mathbb{N})$ denote an infinite sequence of uniformly bounded non-overlapping time intervals with $t_0 = 0$, $t_k - t_{k+1} \geqslant T_d > 0$. T_d is said as the dwell time, during which the graph keeps fixed. The graph changes at switching sequence t_{k+1}. Let $\sigma(t) : [0, \infty) \to \{1, 2, \cdots, h\}$ denote a switching signal. $G_{\sigma(t)}$ and $L_{\sigma(t)}$ represent the graph and Laplacian matrix at t.

Definition 1. *An agent is called a leader if its neighbor set has no agent, otherwise it is called a follower if it has at least one neighbor.*

Lemma 1. *[8] If G is connected, then L has a simple 0 eigenvalue with $1_N \big/ \sqrt{N}$ as its right eigenvector, and all the other eigenvalues are positive.*

2.2 Problem description

Consider a heterogeneous MAS with M leaders and N followers and the system is divided into several groups. The target of the group formation-tracking is that the followers should form the desired sub-formation and track the trajectory of each group leader in the meanwhile.

Assume that there exist $g \in \mathbb{N}\,(g \geqslant 1)$ subgroups and the separation of the nodes for the followers is defined as $V_1, V_2, \cdots, V_g$, which satisfies $V_k \neq \emptyset\,(k = 1, 2, \cdots, g)$, $\cup_{k=1}^{g} V_k = V_F$ and $V_k \cap V_m = \emptyset$ $(k, m \in \{1, 2, \cdots, g\}\,; k \neq m)$. Let $\bar{i}$ as the index of the subgroup to which agent i belong. The number of the followers in subgroup $\bar{i}\,(\bar{i} \in 1, 2, \cdots, g)$ is denoted by $n_F^{\bar{i}}$. Assume that there is only one leader in each subgroup. Therefore, $\sum\limits_{\bar{i}=1}^{g} n_F^{\bar{i}} = N$ and $g = M$. Let $V_{\bar{i}} = \{\Xi_{\bar{i}} + 1, \Xi_{\bar{i}} + 2, \cdots, \Xi_{\bar{i}} + n_{\bar{i}}\}$, where $\Xi_{\bar{i}} = \sum\limits_{k=1}^{\bar{i}-1} n_k$ denotes the index of subgroup $\bar{i}$.

The leader of subgroup $\bar{i}\,(\bar{i} = \{1, 2, \cdots, g\})$ is defined as

$$\left\{\begin{array}{l} \dot{z}_0^{\bar{i}}(t) = S z_0^{\bar{i}}(t) \\ y_0^{\bar{i}}(t) = U z_0^{\bar{i}}(t) \end{array}\right. \tag{1}$$

where $z_0^{\bar{i}}(t) \in \mathbb{R}^q$ and $y_0^{\bar{i}}(t) \in \mathbb{R}^p$ are the state and the output of leader in subgroup $\bar{i}$, $S \in \mathbb{R}^{q\times q}$ and $U \in \mathbb{R}^{p\times q}$ are constant gain matrices. The pair (U, S) is detectable.

The follower of $i\,(i \in \{\Xi_{\bar{i}} + 1, \Xi_{\bar{i}} + 2, \cdots, \Xi_{\bar{i}} + n_{\bar{i}}\})$ in subgroup $\bar{i}\,(\bar{i} = \{1, 2, \cdots, g\})$ can be modeled by

$$\left\{\begin{array}{l} \dot{x}_i^{\bar{i}}(t) = A_i x_i^{\bar{i}}(t) + B_i u_i^{\bar{i}}(t) \\ y_i^{\bar{i}}(t) = C_i x_i^{\bar{i}}(t) \end{array}\right. \tag{2}$$

where $x_i^{\bar{i}}(t) \in \mathbb{R}^{n_i}$, $u_i^{\bar{i}}(t) \in \mathbb{R}^{m_i}$ and $y_i^{\bar{i}}(t) \in \mathbb{R}^p$ are the state, control input and output of the follower i. $A_i \in \mathbb{R}^{n_i\times n_i}$, $B_i \in \mathbb{R}^{n_i\times m_i}$ and $C_i \in \mathbb{R}^{p\times n_i}$ are constant gain known matrices with $\mathrm{rank}(B_i) = m_i$. The system matrixes (A_i, B_i) and (C_i, A_i) are stabilizable and observable, respectively.

Assumption 1. *The following regulator equations:*

$$\left\{\begin{array}{l} E_i S = A_i E_i + B_i F_i \\ 0 = C_i E_i - U \end{array}\right. \tag{3}$$

have solution matrices (E_i, F_i), $i = 1, 2, \cdots, N$.

Remark 1. *Note that Assumption 1 is standard for cooperative control of HMAS. The solvability of regulator equations is important for the output regulation problems.*

The Laplacian matrix $L_{\sigma(t)}$ of the heterogeneous group MASs is shown as follows:

$$\left[\begin{array}{cc} 0_{M\times M} & 0_{M\times N} \\ L_{\sigma(t)}^{EF} & L_{\sigma(t)}^{F} \end{array}\right]$$

Let $L_{\sigma(t)\bar{i}}^{EF}$ represent the communication between the leader and the followers of the subgroup $\bar{i}$, and $L_{\sigma(t)\bar{i}\bar{j}}^{F}$ represent the followers interaction from subgroup $\bar{i}$ to subgroup $\bar{j}$ $(\bar{i}, \bar{j} \in \{1, 2, \cdots, g\})$.

Assumption 2. *For each subgroup $\bar{i}$ ($\bar{i} \in 1, 2, \cdots, g$), the corresponding topology $\bar{G}_{\sigma(t)\bar{i}}$ of the followers is undirected and connected.*

Assumption 3. *For any communication topologies, the sum of each row of $L^F_{\sigma(t)\bar{i}\bar{j}}$, $\bar{i}, \bar{j} \in \{1, 2, \cdots, g\}$ is equal to zero. And the eigenvalues of the $L^F_{\sigma(t)} \in \mathbb{R}^{N\times N}$ are different.*

Let $h_0^{\bar{i}}(t) = \left[h^T_{\Xi_{\bar{i}}+1}(t), h^T_{\Xi_{\bar{i}}+2}(t), \cdots, h^T_{\Xi_{\bar{i}}+n_F^{\bar{i}}}(t)\right]^T \in \mathbb{R}^{nn_i}$ represent the expected formation of subgroup $\bar{i}$ ($\bar{i} \in \{1, 2, \cdots, g\}$), where each component of $h^T_{\Xi_{\bar{i}}+j}(t)$ ($j \in \{\Xi_{\bar{i}}+1, \Xi_{\bar{i}}+2, \cdots, \Xi_{\bar{i}}+n_{\bar{i}}\}$) is piecewise continuously differentiable. Denote $y^{\bar{i}}(t) = \left[y^{\bar{i}T}_{\Xi^{\bar{i}}+1}(t), y^{\bar{i}T}_{\Xi_{\bar{i}}+2}(t)\right.$ $\left., \cdots, y^{\bar{i}T}_{\Xi_{\bar{i}}+n_F^{\bar{i}}}(t)\right]^T \in \mathbb{R}^{nn_{\bar{i}}}$ for the followers of subgroup $\bar{i}$ $\bar{i} \in \{1, 2, \cdots, g\}$.

Definition 2. *If for any given bounded initial values, the HMAS is said to achieve the output group formation tracking for any subgroup $\bar{i}$ ($\bar{i} \in \{1, 2, \cdots, g\}$)*

$$\lim_{t\to 0}\left(y^{\bar{i}}(t) - h_0^{\bar{i}}(t) - \left(1_{n_i} \otimes y_0^{\bar{i}}(t)\right)\right) = 0 \tag{4}$$

The rest of this paper will concentrate on

1) Under what condition the HMASs can realize the desired group formation tracking.
2) How to design the state observer and the output group formation tracking protocol under the influence of time-varying delays and switching interaction topologies.

3 Main results

In this section, state observer for each follower to estimate the group leader is introduced under the influence of both time-varying communication delays and switching interaction topologies. Then, the effectiveness of the proposed observer is to be proved. Furthermore, an observer-based group formation protocol is proposed and an algorithm to determine the constant matrix in the protocol is also put forward.

Consider the following state observer for each follower:

$$\begin{aligned}\dot{\hat{\zeta}}_i^{\bar{i}}(t) = S\hat{\zeta}_i^{\bar{i}}(t) - K_1 \Bigg(& \sum_{j\in N^{Fi}_{\sigma(t)}}^{c=\bar{i}} w_{ij}\left(\hat{\zeta}_i^{\bar{i}}(t-\tau(t))\right. \\ & \left. -\hat{\zeta}_j^{c}(t-\tau(t))\right) + \sum_{j\in N^{Fi}_{\sigma(t)}}^{c\neq\bar{i}} w_{ij}\left(\hat{\zeta}_i^{\bar{i}}(t-\tau(t)) - \hat{\zeta}_j^{c}(t-\tau(t))\right) \\ & - \sum_{k\in N^{Ei}_{\sigma(t)}} w_{ik}\left(\hat{\zeta}_i^{\bar{i}}(t-\tau(t)) - z_0^{\bar{i}}(t-\tau(t))\right)\Bigg)\end{aligned} \tag{5}$$

where $\hat{\zeta}_i^{\bar{i}}(t)$ is the i-th follower's observer belonging to the subgroup $\bar{i}$ ($\bar{i} \in \{1, 2, \cdots, g\}$), and $z_0^{\bar{i}}(t)$ is the leader of subgroup $\bar{i}$. $\tau(t)$ represents the varying delays. K_1 is the constant gain matrix to be calculated later.

Assumption 4. *Varying communication delays $\tau(t)$ satisfies $0 \leqslant \tau(t) \leqslant \sigma$ and $|\dot{\tau}(t)| \leqslant \delta < 1$. σ and δ are known constants, which means $\tau(t)$ is bounded.*

The following lemmas are presented to prove the effectiveness of the proposed observer.

Lemma 2. *[18] A vector-valued function is denoted by $\eta(t) \in \mathbb{R}^{2d}$, whose entries are first-order continuous-derivative. One gets*

$$\begin{aligned}&-\int_{t-\tau(t)}^{t} \dot{\eta}^T(s) P\dot{\eta}(s)ds \\ &\leqslant \gamma^T(t)\begin{bmatrix} X_1^T + X_1 & -X_1^T + X_2 \\ * & -X_2^T - X_1\end{bmatrix}\gamma(t) \\ &+ \tau(t)\gamma^T(t)\begin{bmatrix} X_1^T \\ X_2^T\end{bmatrix} P^{-1}[X_1, X_2]\gamma(t)\end{aligned} \tag{6}$$

where $X_1, X_2 \in \mathbb{R}^{2d}$, $\gamma(t) = \left[\eta^T(t), \eta^T(t-\tau(t))\right]^T$ and P is a positive definite symmetric matrix.

Let $\bar{\lambda}_1 = \min\left\{\lambda^i_{\sigma(t)}\right\}$, $\bar{\lambda}_2 = \max\left\{\lambda^i_{\sigma(t)}\right\}$, $\sigma(t) \in \{1, 2, \cdots, p\}$, $\lambda^i_{\sigma(t)}$ is the eigenvalue of real symmetric positive definite matrix.

Lemma 3. *[10] For any i, the switching signal $\sigma(t) \in \{1, 2, \cdots, p\}$, $\Theta^i_{\sigma(t)} = \Phi_0 + \lambda^i_{\sigma(t)}\Phi_1 < 0$ if and only if $\Theta_i = \Phi_0 + \bar{\lambda}_i\Phi_1 < 0$ ($i \in \{1, 2\}$).*

Lemma 4. *[14] Let $\hat{\zeta}^{\bar{i}}(t) = \left[\hat{\zeta}^{\bar{i}T}_{\Xi_{\bar{i}}+1}(t), \hat{\zeta}^{\bar{i}T}_{\Xi_{\bar{i}}+2}(t), \cdots, \hat{\zeta}^{\bar{i}T}_{\Xi_{\bar{i}}+n_F^{\bar{i}}}(t)\right]^T \in \mathbb{R}^{qn_F^{\bar{i}}}$, and if the state observer satisfies the following equation*

$$\lim_{t\to 0}\left(\hat{\zeta}^{\bar{i}}(t) - (1_F^{n_i} \otimes I_n) z_0^{\bar{i}}(t)\right) = 0 \tag{7}$$

then the observer is said to estimate the leader's state for the subgroup $\bar{i}$.

The constant matrix in state observer K_1 is designed as the following LMI.

If there exist positive symmetric matrices R, Ω, X and real matrix $\bar{K}_1$, LMI (8) is feasible for any $\bar{\lambda}_i^F$ ($i = 1, 2$)

$$\prod(\bar{\lambda}_i) = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & 0 & R \\ * & \Xi_{22} & \Xi_{23} & \sigma X & 0 \\ * & * & -\sigma X & 0 & 0 \\ * & * & * & -\sigma X & 0 \\ * & * & * & * & -\Omega \end{bmatrix} < 0 \tag{8}$$

where

$$\begin{aligned}\Xi_{11} &= RS^T + SR - \bar{\lambda}_i^F\bar{K}_1 - \bar{\lambda}_i^F\bar{K}_1^T - (1-\delta)\Omega \\ \Xi_{12} &= R - \bar{\lambda}_i^F\bar{K}_1 - (2-\delta)\Omega \\ \Xi_{13} &= \sigma RS^T - \sigma\bar{\lambda}_i^F\bar{K}_1^T \\ \Xi_{22} &= -(3-\delta)\Omega \\ \Xi_{23} &= -\sigma\bar{\lambda}_i^F\bar{K}_1^T\end{aligned}$$

$\bar{\lambda}_i^F$ ($i = 1, 2$) are the minimum and maximum eigenvalues of the followers Laplacian matrix $L^F_{\sigma(t)}$.

Then the gain matric can be defined as $K_1 = \bar{K}_1\Omega^{-1}$.

Based on the calculated K_1, the following Theorem can be derived.

Theorem 1. *The proposed state observer can estimate the leader state for each subgroup under the influence of both time-varying communication delays and switching interaction topologies.*

Proof. Let $\tilde{\zeta}^{\bar{i}}(t) = \hat{\zeta}^{\bar{i}}(t) - \left(1_{n_F^{\bar{i}}} \otimes I_q\right) z_0^{\bar{i}}(t)$ and $\tilde{\zeta}^{i}(t) = \left[\tilde{\zeta}^{1T}(t), \tilde{\zeta}^{2T}(t), \cdots, \hat{\zeta}^{gT}(t)\right]^T \in \mathbb{R}^{Nq}$. Then, one gets

$$\dot{\tilde{\zeta}}(t) = (I_N \otimes S)\tilde{\zeta}(t) - \left(L^F_{\sigma(t)} \otimes K_1\right)\tilde{\zeta}(t-\tau(t)) \tag{9}$$

Considering the following common Lyapunov-Krasovskii candidate function:

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{10}$$

where
$V_1(t) = \tilde{\zeta}^T(t)\left(I_N \otimes R^{-1}\right)\tilde{\zeta}(t)$,
$V_2(t) = \int_{t-\tau(t)}^{t} \tilde{\zeta}^T(s)\left(I_N \otimes \Omega^{-1}\right)\tilde{\zeta}(s)\,ds$,
$V_3(t) = \int_{-\sigma}^{0}\int_{t+\mu}^{t} \dot{\tilde{\zeta}}^T(s)\left(I_N \otimes X^{-1}\right)\dot{\tilde{\zeta}}(s)\,ds\,d\mu$.

Based on the Assumption 3, Let $\Lambda^F_{\sigma(t)} = diag\left(\lambda^1_{\sigma(t)}, \lambda^2_{\sigma(t)}, \cdots, \lambda^N_{\sigma(t)}\right)$, then there exists an orthogonal matrix $M_{\sigma(t)} \in \mathbb{R}^{N\times N}$ satisfying $M^T_{\sigma(t)} L^F_{\sigma(t)} M_{\sigma(t)} = \Lambda^F_{\sigma(t)}$.

Define $\eta(t) = \left(M^T_{\sigma(t)} \otimes I_{Nn}\right)\tilde{\zeta}(t) = \left[\eta_1^T(t), \eta_2^T(t), \cdots, \eta_N^T(t)\right]^T$, take the derivative of $V(t)$ along the (10)

$$\dot{V}_1(t) = \sum_{i=1}^{N} \hat{\eta}_i^T(t)\begin{bmatrix} R^{-1}S + S^T R^{-1} & -\lambda^i_{\sigma(t)} R^{-1} B K_1 \\ * & 0 \end{bmatrix}\hat{\eta}_i(t) \tag{11}$$

where $\hat{\eta}_i(t) = \left[\eta_i^T(t), \eta_i^T(t-\tau(t))\right]^T$.

Based on Assumption 3, $\dot{V}_2(t)$ can be written as

$$\begin{aligned} \dot{V}_2(t) &\le \eta^T(t)\left(I_N \otimes \Omega^{-1}\right)\eta(t) \\ &\quad - (1-\delta)\,\eta^T(t-\tau(t))\left(I_N \otimes \Omega^{-1}\right)\eta(t-\tau(t)) \\ &= \sum_{i=1}^{N} \hat{\eta}_i^T(t)\begin{bmatrix} \Omega^{-1} & 0 \\ 0 & -(1-\delta)\Omega^{-1} \end{bmatrix}\hat{\eta}_i(t) \end{aligned} \tag{12}$$

$$\begin{aligned} \dot{V}_3(t) &= \sigma\dot{\eta}^T(t)\left(I_N \otimes X^{-1}\right)\dot{\eta}(t) \\ &\quad - \int_{t-\sigma}^{t} \dot{\eta}^T(s)\left(I_N \otimes X^{-1}\right)\dot{\eta}(s)\,ds \end{aligned} \tag{13}$$

Let $\varpi_i = \left[S\,,\, -\lambda^i_{\sigma(t)} B K_1\right]$, the first half of the equation (25) is given as

$$\sigma\dot{\eta}^T(t)\left(I_N \otimes X^{-1}\right)\dot{\eta}(t) = \sigma\sum_{i=1}^{N}\hat{\eta}_i^T(t)\,\varpi_i^T X^{-1}\varpi_i\hat{\eta}_i(t) \tag{14}$$

From Assumption 4 and Lemma 2, the latter part is given as

$$\begin{aligned} &-\int_{t-\sigma}^{t}\dot{\eta}^T(s)\left(I_N \otimes X^{-1}\right)\dot{\eta}(s)\,ds \\ &\le \sum_{i=1}^{N}\hat{\eta}_i^T(t)\left(\begin{bmatrix} M_1^T + M_1 & -M_1^T + M_2 \\ * & -M_2^T - M_2 \end{bmatrix}\right. \\ &\quad \left. +\sigma\begin{bmatrix} M_1^T \\ M_2^T \end{bmatrix} X^{-1}\left[M_1, M_2\right]\right)\hat{\eta}_i(t) \end{aligned} \tag{15}$$

Define $M_1 = -R^{-1}, M_2 = \Omega^{-1}$, form (10) to (15), ones get

$$\dot{V}(t) \leqslant \sum_{i=1}^{N}\hat{\eta}_i^T(t)\mathrm{Z}_i\hat{\eta}_i(t) \tag{16}$$

where
$\mathrm{Z}_i = \mathrm{T}_i + \sigma\varpi_i^T X^{-1}\varpi_i + \sigma\begin{bmatrix} -R^{-T} \\ \Omega^{-T} \end{bmatrix} X^{-1}\left[-R^{-1}, \Omega^{-1}\right]$,
$\mathrm{T}_i = \begin{bmatrix} \mathrm{T}_{i11} & \Omega^{-1} + R^{-1} - \lambda^i_{\sigma(t)} R^{-1} B K_1 \\ * & -(3-\delta)\,\Omega^{-1} \end{bmatrix}$,
$\mathrm{T}_{i11} = -2R^{-1} + R^{-1}S + S^T R^{-1} + \Omega^{-1}$.

It can be verified by Schur complement lemma, $\mathrm{Z}_i < 0$ is equivalent to $\Psi_i < 0$

$$\Psi_i = \begin{bmatrix} T_i & \sigma\varpi_i^T & \sigma\begin{bmatrix} -R^{-1} & -\Omega^{-1} \end{bmatrix} \\ * & \sigma S^{-1} & 0 \\ * & * & -\sigma S^{-1} \end{bmatrix} < 0$$

Choosing $\Gamma = \begin{bmatrix} R & 0 \\ \Omega & \Omega \end{bmatrix}$ and $\bar{\Gamma} = diag\{T, I, X\}$, then one gets

$$\bar{\Gamma}^T\psi_i\bar{\Gamma} = \begin{bmatrix} \Gamma^T T\Gamma_i & \sigma\Gamma^T\varpi_i^T & \sigma\begin{bmatrix} 0 & X \end{bmatrix} \\ * & \sigma X & 0 \\ * & * & -\sigma X \end{bmatrix}$$

Based on the calculated $K_1 = \bar{K}_1\Omega^{-1}$ and Lemma 2, $\prod\left(\bar{\lambda}_i\right) < 0$ are equivalent to $\prod\left(\lambda^i_{\sigma(t)}\right) < 0$ $(i = 2, 3, \cdots, N, \sigma(t) = 1, 2, \cdots, p)$. Then according to Schur complement lemma, $\prod\left(\lambda^i_{\sigma(t)}\right) < 0$ if and only if $\bar{\Gamma}^T\psi_i\bar{\Gamma} < 0$. One gets

$$\lim_{t\to\infty}\tilde{\zeta}(t) = 0 \tag{17}$$

Therefore observer's error $\tilde{\zeta}(t)$ converges to zero as $t \to \infty$ with both communication delays and switching interaction topologies. This completes the proof.

□

Consider the following observer-based group formation-tracking protocol for follower i $(i \in \{\Xi_{\bar{i}} + 1, \Xi_{\bar{i}} + 2, \cdots, \Xi_{\bar{i}} + n_{\bar{i}}\})$ in subgroup $\bar{i}$ $(\bar{i} = \{1, 2, \cdots, g\})$

$$u_i^{\bar{i}}(t) = K_{2i}x_i^{\bar{i}}(t) + K_{3i}\left(\hat{\zeta}_i^{\bar{i}}(t) + h_i^{\bar{i}}(t)\right) + r_i^{\bar{i}}(t) \tag{18}$$

where K_{2i} and K_{3i} are the constant matrices, $r_i^{\bar{i}}(t) \in \mathbb{R}^{m_i}$ is the compensation input for the group formation tracking.

Note that $\text{rank}(B_i) = m_i$, there exists a nonsingular matrix $\Gamma_i = \left[\hat{B}_i^T, \tilde{B}_i^T\right]^T$ with $\hat{B}_i \in \mathbb{R}^{(n_i - m_i)\times n_i}$ and $\tilde{B}_i \in \mathbb{R}^{(n_i - m_i)\times n_i}$.

Algorithm 1: Steps of designing the gain matrices.

Step 1: Solve the regulator for the pair (E_i, F_i).

Step 2: For the given formation, check the following feasibility condition

$$\lim_{t\to\infty}\left(\tilde{B}_i E_i\left(Sh_i^{\bar{i}}(t) - \dot{h}_i^{\bar{i}}(t)\right)\right) = 0 \tag{19}$$

Step 3: Calculate the compensation input $r_i^{\bar{i}}(t)$ as follows

$$r_i^{\bar{i}}(t) = -\hat{B}_i E_i\left(Sh_i^{\bar{i}}(t) - \dot{h}_i^{\bar{i}}(t)\right) \tag{20}$$

Step 4: The gain metric K_1 in the distributed observer (4) can be given by LMI.

Step 5: Chose appropriate K_{2i} such that $A_i + B_iK_{2i}$ is Hurwitz, and $K_{3i} = F_i - K_{2i}E_i$.

Theorem 2. *If there exist formations satisfying (19), HMAS is said to achieve the group formation tracking with time-varying delays and switching interaction topologies under the protocol designed by Algorithm 1.*

Proof. Based on protocol (18), the followers' systems can be written as

$$\begin{aligned}\dot{x}_i^{\bar{i}}(t) = (A_i{+}B_iK_{2i})\, x_i^{\bar{i}}(t) + B_iK_{3i}\left(\hat{\zeta}_i^{\bar{i}}(t) + h_i^{\bar{i}}(t)\right) \\ +Br_i^{\bar{i}}(t)\end{aligned} \tag{21}$$

The group formation-tracking error of follower i in subgroup $\bar{i}$ denotes by $\phi_i^{\bar{i}}(t) = x_i^{\bar{i}}(t) - h_i^{\bar{i}}(t) - \zeta_i^{\bar{i}}(t)$.

From Algorithm 1, one gets

$$\begin{aligned}\dot{\phi}_i^{\bar{i}}(t) = (A_i + B_iK_{2i})\,\phi_i^{\bar{i}}(t) + B_iK_{3i}\tilde{\zeta}_i^{\bar{i}}(t) \\ +SE_ih_i^{\bar{i}}(t) - E_i\dot{h}_i^{\bar{i}}(t) + B_ir_i^{\bar{i}}(t)\end{aligned} \tag{22}$$

Because the formation feasibility conditions are satisfied, one has

$$\lim_{t\to\infty}\left(\tilde{B}_iE_i\left(Sh_i^{\bar{i}}(t) - \dot{h}_i^{\bar{i}}(t)\right) + \tilde{B}_iB_ir_i^{\bar{i}}(t)\right) = 0 \tag{23}$$

Based on (20), it can be obtained that

$$\hat{B}_iE_i\left(Sh_i^{\bar{i}}(t) - \dot{h}_i^{\bar{i}}(t)\right) + \hat{B}_iB_ir_i^{\bar{i}}(t) = 0 \tag{24}$$

Note that $\Gamma_i = \left[\hat{B}_i^T, \tilde{B}_i^T\right]^T$ is nonsingular. It can be verified from (23) and (24) that

$$\lim_{t\to\infty}\left(SE_ih_i^{\bar{i}}(t) - E_i\dot{h}_i^{\bar{i}}(t) + B_ir_i^{\bar{i}}(t)\right) = 0 \tag{25}$$

Note that based on Theorem 1 $\lim_{t\to 0}\left(\zeta_i^{\bar{i}}(t)\right) = 0$ and $A_i + B_iK_{2i}$ is Hurwitz, thus $\lim_{t\to 0}\left(\phi_i^{\bar{i}}(t)\right) = 0$, which means that HMASs can realize the group formation tracking under the influence of both time delays and switching topologies. This completes the proof of Theorem 2. □

Remark 2. *According to Theorem 2, only the formation satisfying the formation feasibility condition can be realized. And based on the protocol, the group formation tracking problem in a more complex environment is proven to be achieved.*

4 Numerical simulations

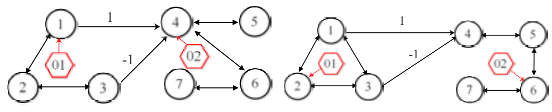
In this section, an illustrative simulation is shown to verify the effectiveness of the proposed protocol and algorithm.

Consider a HMAS with nine agents and divided into two subgroups. Let $V_1 = \{1,2,3\}$ and $V_2 = \{4,5,6,7\}$ denote the followers of each subgroup. The number of the followers in each group are defined as $n_F^1 = 3$, $n_F^2 = 4$. Let $\tau(t) = 0.06 + 0.02\sin(t)$. The switching interaction topologies are shown in the Fig. 1.

The dynamic of each leader is shown as:

$$S=\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}, U=\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The dynamics of the followers are presented with:



(a) G_1 (b) G_2

Fig. 1: Switching topologies

$$A_{\bar{i}}=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & -2 & -1 \end{bmatrix}, B_{\bar{i}}=\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, C_{\bar{i}}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$E_i = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, F_i = \begin{bmatrix} 0 & -1 & -1 \\ 2 & -2 & 0 \end{bmatrix}. (i = 1,2,3)$$

$$A_j = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -1 \end{bmatrix}, \quad B_j = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix},$$

$$C_j = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_j = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad F_j = \begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & -1 \end{bmatrix}. (j = 4,5,6,7)$$

The desired formations for each group are shown as follow

$$h_i^{\bar{i}}(t) = \begin{bmatrix} r_{\bar{i}}\sin\left(t + \frac{(n_F^{\bar{i}}-1)2\pi}{n_F^{\bar{i}}}\right) \\ -r_{\bar{i}}\cos\left(t + \frac{(n_F^{\bar{i}}-1)2\pi}{n_F^{\bar{i}}}\right) \\ r_{\bar{i}}\cos\left(t + \frac{(n_F^{\bar{i}}-1)2\pi}{n_F^{\bar{i}}}\right) \end{bmatrix}$$

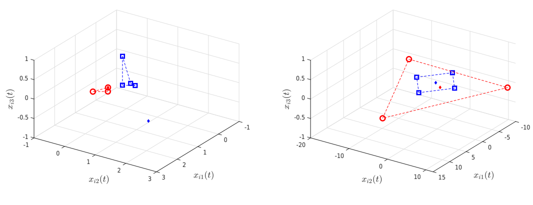
where $\bar{i} = \{1,2\}$, $r_1 = 15m$ and $r_2 = 5m$. It can be obtained that the formation tracking feasibility conditions are satisfied. The formation compensation inputs are given as $r_i^{\bar{i}}(t) = 0$.

The gain matrixes are given as follows:

$$K_1 = \begin{bmatrix} 0.4880 & \text{-}0.0148 & 0 \\ 0.0148 & 0.4880 & 0 \\ 0 & -0 & \text{-}0.0696 \end{bmatrix}, \quad K_{2i} = \begin{bmatrix} -6 & -5 & -1 \\ -2 & 2 & 0 \end{bmatrix}, \quad K_{3i} = \begin{bmatrix} 5 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad K_{2j} = \begin{bmatrix} 1 & 2 & -1 \\ -12 & -7 & -1 \end{bmatrix}, \quad K_{3j} = \begin{bmatrix} 0 & 0 & 1 \\ 7 & 11 & 0 \end{bmatrix}.$$

$(i = 1,2,3; j = 4,5,6,7)$

The initial states of the leaders and followers are chosen as $x_i^{\bar{i}}(0) = 2(\Theta - 0.5)\,(i = \{1,2,\cdots,7\}; \bar{i} = \{1,2\})$ and $z_0^{\bar{i}}(0) = 2(\Theta - 0.5)\,(\bar{i} = \{1,2\})$, where Θ is a pseudo-random value that satisfies the uniform distribution between $(0,1)$. The initial value of the observes are zero.



(a) $t = 0s$ (b) $t = 47s$

Fig. 2: Snapshots of seven agents ($t = 0s; t = 47s$)

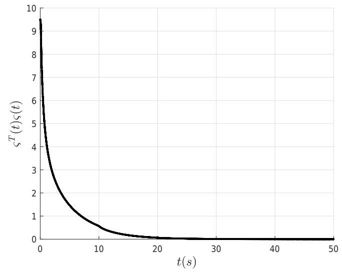


Fig. 3: State observers' error within $t = 50s$

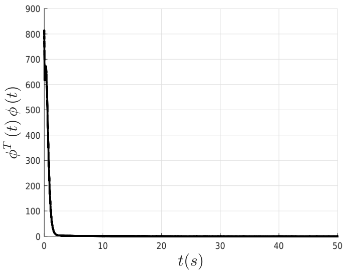


Fig. 4: Group formation tracking error within $t = 50s$

Fig. 2 shows the state snapshots of nine agents with $t = 0s$ and $47s$, respectively. Each group formation is denoted by different color. Fig. 3 denotes that the state observers' error can converge to zero within $t = 50s$. In Fig. 4, group formation tracking error is also convergent, therefore, the H-MASs can achieve the group formation-tracking.

5 Conclusions

Time-varying group formation tracking problems for H-MASs under the influence of both communication delays and switching topologies are investigated in this paper. To solve the multiple communication constraints, a distributed observer is proposed to evaluate the state of leader. Then, an observer-based protocol is given for each follower. Moreover, the convergence of the group-formation tracking is also presented, which means that the HMASs can realize the group-formation tracking with both varying time-delays and switching network. Future research will concentrate on the group formation-tracking control problem for HMASs without well-informed follower

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