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Predefined containment control for general linear multiagent systems with time-varying delays and switching topologies

Shiyu Zhou[1] | **Yongzhao Hua**[1] | **Xiwang Dong**[1,2] | **Qingdong Li**[1] | **Zhang Ren**[1,2]

[1]School of Automation Science and Electrical Engineering, Beihang University, Beijing, China

[2]Beijing Advanced Innovation Center for Big Data and Brain Computing, Beihang University, Beijing, China

Correspondence
Xiwang Dong, School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, China.
Email: xwdong@buaa.edu.cn

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Abstract

Containment control problems for general linear multiagent systems with switching interaction topologies and time-varying delays are studied. Firstly, state observers are constructed to estimate the multiple leaders under the influence of switching graphs and time-varying delays. Secondly, a predefined containment protocol is presented based on the distributed state observer, where the expected convex combination of multiple leaders is predefined by several given weights. Moreover, an algorithm with two steps to confirm the gain matrices of the containment protocol and state observer is given. Based on linear matrix inequality technique and common Lyapunov-Krasovskii stability theory, the convergence of this control strategy for multiagent systems with switching topologies and time-varying delays to achieve state containment is proved. Finally, a numerical simulation is given to verify the validity of the theoretical results.

KEYWORDS

containment control, general-linear multiagent systems, switching topologies, time-varying delays

1 | INTRODUCTION

In the last few years, cooperative control of multiagent systems (MASs) has attracted a lot of attention because of its great robustness, low cost, and high performance. The cooperation strategies have been widely employed in different fields such as unmanned aerial vehicles, satellites formation, unmanned ground vehicles, and so on.[1-4] The existing cooperation results could be divided into consensus control,[5] formation control, containment control, and formation-containment control. As a fundament issue for MASs, the target of consensus control is to ensure all MASs can achieve an agreement.[6,7] If there exists one leader, the consensus problem can be categorized into the single-leader consensus tracking.[8] However,

Abbreviations: MASs, multiagent systems; LMI, linear matrix inequality technique.

when multiple leaders are present, containment control arises. The goal of containment control is that followers are allowed to enter the convex hull formed by the multiple leaders.[9-11]

In fact, containment control has been investigated during the past decades, and many fruitful results have been achieved. As the pioneering work on containment control problem, Ji et al[12] firstly proposed a hybrid control strategy based on stop-go rules for first-order MASs to analyze containment conditions under fixed and undirected topology. Cao and Liu had demonstrated necessary and sufficient conditions for the first-order and second-order MASs to realize containment.[10,13] A containment control method for second-order MASs without velocity measurements was studied in the works of Zhou et al[14] and Xu et al,[15] respectively. In order to have a better application, Wen et al[16] proposed a new class of distributed observer-type containment protocols based only on the relative output measurements of the neighboring agents. Qin et al[17] had investigated the output containment control problem for a network of heterogeneous linear MASs. By discussing the relationship between the network structure and some systems parameters, Liu and Su[18] have proposed novel containment protocols via intermittent sampled position data communication.

It should be pointed out that all the above containment results only suppose that there are no time-delay and switching topologies. However, in some practical applications, due to the congestion of the communication channel, time delays may occur. In the meantime, the communication topology between the MASs may be switching due to the failure of communication equipment and the new communication connections. The time delay will affect the control performance of the system. The switching topologies may change the connectivity of the system and the interaction between the agents, resulting the unexpected instabilities of the systems. Hence, a containment control problem with time-varying delays and switching interaction topologies is taken into consideration.

Motivated by this observation, time delay regarded as a common phenomenon has been widely studied in the field of MASs.[19-21] The effect of time-varying delays on MASs was analyzed in the work of Liu et al.[22] Considering uncertainties and communication delays, by utilizing the state information of each agent and neighboring agents, Dong[11] constructed a protocol for general linear MASs with time-varying delays. Necessary and sufficient conditions for containment control of discrete-time MASs with time delay were studied.[23] The solutions varied from frequency domain analysis, properties of SIA matrix, Lyapunov function method, etc. Then, the switching topology case for consensus problems has been extensively studied.[7,24] Notarstefano and Cao discussed the containment control for first and second swarm systems under the influence of switching topologies in the works of Cao et al[13] and Notarstefano et al,[25] respectively. In the work of Lou and Hong,[26] containment control for MASs with stochastic topologies was investigated. Hua et al[27] had proposed a predefined containment control protocol based on a distributed observer to solve totally heterogeneous linear MASs on switching graphs.

However, time-varying delays and switching interaction topologies are rarely considered simultaneously. However, in the work of Liu and Xie,[28] some sampling data-based protocols were proposed to solve the containment problem for second-order MASs under switching topologies and time delays, and Wang et al[29] proposed some sufficient conditions for second-order MASs with dynamically switching topologies and communication delays. When it comes to general linear MASs, it is difficult to solve the containment problem using the above strategies.

Inspired by the aforementioned works and study, in this research, a novel containment control approach for general linear MASs with time-varying delays and switching topologies is designed. First, state observers are constructed under the influence of switching graphs and time-varying delays. Then, a containment protocol is presented based on the distributed state observer, where the expected convex of multiple leaders is predefined by several given weights. Therefore, the switching interaction topologies will not influence the desired targets of followers. Moreover, an algorithm with two steps to determine the gain matrices of the containment protocol and state observer is given. Based on a linear matrix inequality (LMI) technique and common Lyapunov-Krasovskii stability theory, the convergence of the proposed state observer and the control protocol for MASs with switching topologies and time-varying delays to realize state containment is proved. Finally, a numerical simulation is provided to testify the effectiveness of the theoretical results.

Compared with previous works on containment control problems, this paper has two major contributions. First, the effects of time-varying delays and switching interaction topologies are considered for general linear MASs in containment control, simultaneously. In other works, switching interaction topologies[19-21] and time delays[7,24,30] are studied. Second, a distributed state observer is given to handle the influences of both switching graphs and time-varying delays. A predefined containment control protocol is presented for the followers in this research, where the convex combinations of the multiple leaders have no relation to the graphs and can be designated arbitrarily.

The rest of this paper is organized as follows. In Section 2, basic concepts and some results on graph theory are introduced and the problem formulation is given. The main results are provided in Section 3. Simulation results are shown in Section 4. Finally, Section 5 concludes the whole work.

Throughout this paper, I represents an identity matrix with appropriate size. $\otimes$ denotes Kronecker product. $\mathbf{1}$ is used to describe a column vector with 1 as its element. $\text{diag}\{D_1, \dots, D_N\}$ is applied to represent a block diagonal matrix, with $D_i \in \mathbb{R}^{p_i \times q_i} (i = 1, 2, \dots, N)$ being diagonal elements.

2 | PRELIMINARIES AND PROBLEM DESCRIPTION

In this section, some basic results on graph theory are demonstrated, and the problem in this paper is presented.

2.1 | Basic results on graph theory

The weighted graph $G = \{V, T, W\}$ is used to demonstrate the interaction topology among agents, where $V = \{v_1, v_2, \dots v_N\}$ is the vertex set, $T \subseteq \{(v_i, v_j) : v_i, v_j \in V\}$ represents the set of edges, and $W = [a_{ij}] \in \mathbb{R}^{N \times N}$ is the adjacency matrix with nonnegative elements of G. Denote $e_{ij} = (v_i, v_j)$ by the edge in G. Define $w_{ij} > 0$ if $e_{ji} \in T$; otherwise, $w_{ij} = 0$. $N_i = \{v_j \in V : (v_j, v_i) \in T\}$ is the set of neighbors of node v_i. The Laplacian matrix of G is defined as $L = D - W$, where D is a diagonal matrix defined as $D = \text{diag}\{\sum_{j=1}^{N} w_{1j}, \sum_{j=1}^{N} w_{2j}, \dots, \sum_{j=1}^{N} w_{Nj}\}$.

A series of ordered edges $(v_{i1}, v_{i2}), (v_{i2}, v_{i3}), \dots, (v_{ik-1}, v_{ik})$ is defined as a path between v_{i1} and v_{ik}. The definition of undirected graph is that if $e_{ij} \in T$ implies $e_{ji} \in T$ and $w_{ij} = w_{ji}$. If there exists a path from each node to any other node, the undirected graph is said to be connected. The following lemma is useful for analyzing the containment problem of MASs.

Lemma 1 (See the work of Olfati-Saber and Murray[6]). *If G is said to be connected, then L has a simple 0 eigenvalue with $1_N/\sqrt{N}$ as its right eigenvector, and all the other eigenvalues are positive.*

Definition 1. An agent is called a leader if it has no neighbors and a follower if it has at least a neighbor.

The index set of all the switching graphs are represented by $\text{H} = \{1, 2, \dots, h\}$. There is an infinite sequence of nonoverlapping time intervals $[t_k, t_{k+1})$ with $t_0 = 0$, $t_k - t_{k+1} \geq T_d > 0$. T_d is said as the dwell time, during which the graph remains the same. The graph changes at switching sequence t_{k+1}. Let $\sigma(t) : [0, \infty) \to \{1, 2, \dots, h\}$. denotes a switching signal. $G_{\sigma(t)}$ and $L_{\sigma(t)}$ represent the graph and Laplacian matrix at t. Let $G^F_{\sigma(t)}$ and $L^F_{\sigma(t)}$ denote the graph and Laplacian matrix among the followers at t.

Assumption 1. Each switching topology $G^F_{\sigma(t)}$ among the followers is connected and undirected.

Assumption 2. There is at least one follower that has access to all leaders' states at each possible graph.

2.2 | Problem description

Consider a MAS with M leaders and N followers. The interaction topology of the MAS can be described as G. The interaction channel can be denoted by e_{ij}, and w_{ij} represents the interaction strength. Let $O_L = \{1, 2, \dots, M\}$ and $O_F = \{M+1, M+2, \dots, M+N\}$ represent the leader set and follower set, respectively.

Then, the dynamic of the follower and leader of MAS can be expressed as

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_i(t) & i \in O_F \\ \dot{z}_i(t) = Az_i(t) & i \in O_L, \end{cases} \tag{1}$$

where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $x_i(t) \in \mathbb{R}^n$, and $u_i(t) \in \mathbb{R}^n$ are the state and the control input of the ith follower, and $z_i(t) \in \mathbb{R}^n$ is the state of the ith leader.

Definition 2. The multiagent system described by (1) is said to realize state containment if there exist nonnegative constants $\rho_{ij} (i \in O_F, j \in O_L)$ satisfying $\sum_{j=1}^{M} \rho_{ij} = 1$, underlying any bounded initial states, such that

$$\lim_{t \to \infty} \left(x_i(t) - \sum\nolimits_{j=1}^{M} \rho_{ij} z_j(t) \right) = 0. \tag{2}$$

Consider the following state containment protocol and observer of the ith follower with time-varying delays and switching topologies:

$$u_i(t) = K_1\left(x_i(t) - \sum_{j=1}^{M}\rho_{ij}\hat{\xi}_{i,j}(t)\right) \tag{3}$$

$$\dot{\hat{\xi}}_i(t) = (I_M \otimes A)\,\hat{\xi}_i(t) - K_2\left[b_i^{\sigma(t)}\left(\hat{\xi}_i(t-\tau(t)) - z(t-\tau(t))\right) + \sum_{k=M+1}^{M+N} w_{ik}\left(\hat{\xi}_i(t-\tau(t)) - \hat{\xi}_k(t-\tau(t))\right)\right], \tag{4}$$

where $i = 1, 2, \dots, N$, $\hat{\xi}_i(t) = [\hat{\xi}_{i,1}^T(t), \hat{\xi}_{i,2}^T(t), \dots, \hat{\xi}_{i,M}^T(t)]^T$ with $\hat{\xi}_{i,j}(t)$ representing the ith follower's estimate for the state of the jth leader. $z(t) = [z_1^T(t), z_2^T(t), \dots, z_M^T(t)]^T$. K_1 and K_2 are the constant gain matrices. ρ_{ij} is the predefined nonnegative constant satisfying $\sum_{j=1}^{M}\rho_{ij} = 1$, and the nonnegative $b_i^{\sigma(t)} > 0$ if and only if follower i has access to all leaders' states at t; otherwise, $b_i^{\sigma(t)} = 0$. $\tau(t)$ is the time-varying delay that satisfies the following assumption. Because all agents have the same control chip and the systems are isomorphic, it makes sense that $\tau(t)$ is the same.

Assumption 3. $0 \le \tau(t) \le \sigma$ and $|\dot{\tau}(t)| \le \delta < 1$, where σ and δ are known constants.

This paper mainly concentrates on the following two problems for general linear MAS: (1) how to design the protocol and observer for MAS and (2) under what condition can the state containment control be realized.

3 | MAIN RESULTS

In this section, firstly, the containment problems for general-linear MAS with time-varying delays and switching interaction topologies are analyzed. Secondly, an algorithm to calculate the gain matrices in the protocol and state observer is proposed. Finally, using Lyapunov-Krasovskii theory and LMI technique, the stability of control protocol and the convergence of observer are proved.

Under protocol (3) and observer (4), the closed-loop of MAS can be written as

$$\begin{cases} \dot{x}_i(t) = (A + BK_1)x_i(t) - BK_1\sum_{j=1}^{M}\rho_{ij}\hat{\xi}_{i,j}(t) & i \in O_F \\ \dot{z}_i(t) = Az_i(t) & i \in O_L. \end{cases} \tag{5}$$

Let the containment error be $\tilde{x}_i(t) = x_i(t) - \sum_{j=1}^{M}\rho_{ij}z_j(t)$. Based on (5), it could be written as

$$\dot{\tilde{x}}_i(t) = (A + BK_1)x_i(t) - BK_1\sum_{j=1}^{M}\rho_{ij}\hat{\xi}_{i,j}(t) - A\sum_{j=1}^{M}\rho_{ij}z_j(t). \tag{6}$$

Let $\tilde{\xi}_{i,j}(t) = \hat{\xi}_{i,j}(t) - z_j(t)$ denote the observing error between the ith observer and the jth leader. Then, (6) could be rewritten as follows:

$$\dot{\tilde{x}}_i(t) = (A + BK_1)\tilde{x}_i(t) - BK_1\sum_{j=1}^{M}\rho_{ij}\tilde{\xi}_{i,j}(t). \tag{7}$$

Let $z(t) = [z_1^T(t), z_2^T(t), \dots, z_M^T(t)]^T$ and $\tilde{\xi}_i(t) = \hat{\xi}_i(t) - z(t)$. It can be verified from (4) that

$$\dot{\tilde{\xi}}_i(t) = (I_M \otimes A)\tilde{\xi}_i(t) - K_2\left[b_i^{\sigma(t)}\tilde{\xi}_i(t-\tau(t)) + \sum_{k=M+1}^{M+N}\omega_{ik}\left(\tilde{\xi}_i(t-\tau(t)) - \tilde{\xi}_k(t-\tau(t))\right)\right]. \tag{8}$$

Let $\varsigma(t) = [\tilde{\xi}_1^T(t), \tilde{\xi}_2^T(t), \dots, \tilde{\xi}_N^T(t)]^T$ and $\tilde{A} = I_M \otimes A$; then, (8) could be rewritten as follows:

$$\dot{\varsigma}(t) = (I_N \otimes \tilde{A})\varsigma(t) - (H_{\sigma(t)} \otimes K_2)\varsigma(t-\tau(t)), \tag{9}$$

where $H_{\sigma(t)} = B_{\sigma(t)} + L_F^{\sigma(t)}$ with $L_F^{\sigma(t)}$ being followers' symmetric Laplacian matrix at t, and $B_{\sigma(t)} = \mathrm{diag}[b_1^{\sigma(t)}, b_2^{\sigma(t)}, \dots, b_N^{\sigma(t)}]$. The next lemma shows a useful property of $H_{\sigma(t)}$.

Lemma 2. *If each switching topology $G^F_{\sigma(t)}$ is connected, then the symmetric matric $H_{\sigma(t)}$ at t associated with $G^F_{\sigma(t)}$ is positive definite.*

Proof. Let $\lambda_1^{\sigma(t)}, \dots, \lambda_N^{\sigma(t)}$ with $\lambda_1^{\sigma(t)} < \lambda_2^{\sigma(t)} \le \dots \le \lambda_N^{\sigma(t)}$ and $\zeta_1^{\sigma(t)}, \dots, \zeta_N^{\sigma(t)}$ be the eigenvalues and eigenvectors of followers' Laplacian matrix $L_F^{\sigma(t)}$ at t, respectively. From Lemma 1, $\lambda_1^{\sigma(t)} = 0; \lambda_i^{\sigma(t)} > 0, i \geqslant 2$; and $\zeta_1^{\sigma(t)} = 1$ is an eigenvector corresponding to $\lambda_1^{\sigma(t)}$. Then, for any nonzero vector $z^{\sigma(t)} \in \mathbb{R}^{\mathbb{N}}$, there exists $z^{\sigma(t)} = \sum_{i=1}^{N} c_i^{\sigma(t)}\zeta_i^{\sigma(t)}$, where $c_i^{\sigma(t)}$ $(i \in O_F)$ are some constants. Moreover, from Assumption 2, $B_{\sigma(t)} \neq 0$ because there is at least one follower that has access to all leaders' states. We provide $b_j^{\sigma(t)} > 0$ for some j, and there is no doubt that $(\zeta_1^{\sigma(t)})^T B_{\sigma(t)}(\zeta_1^{\sigma(t)}) \ge b_j^{\sigma(t)}$. Consequently, the case is considered when $c_2^{\sigma(t)} = \dots = c_N^{\sigma(t)} = 0$ (so $c_1^{\sigma(t)} \neq 0$) or when $c_i^{\sigma(t)} \neq 0$ $(i \ge 2)$, and it can be obtained that

$$(z^{\sigma(t)})^T H_{\sigma(t)}(z^{\sigma(t)}) = (z^{\sigma(t)})^T B_{\sigma(t)}(z^{\sigma(t)}) + (z^{\sigma(t)})^T L_F^{\sigma}(z^{\sigma(t)}) \ge \sum_{i=2}^{n} \lambda_i^{\sigma(t)}\left(c_i^{\sigma(t)}\right)^2\left(\zeta_i^{\sigma(t)}\right)^T \zeta(z^{\sigma(t)})^T B_{\sigma(t)}(z^{\sigma(t)}) > 0. \tag{10}$$

For $z \neq 0$, the conclusion has been proved. □

The following lemmas are help to analyze the state containment problem with time-varying delays and switching topologies.

Lemma 3 (See the work of Zhang et al[31]). *Let $\eta(t) \in \mathbb{R}^{2d}$ be a vector-valued function with first-order continuous-derivative entries. The following integral inequality holds:*

$$-\int_{t-\tau(t)}^{t} \dot{\eta}^T(s)S\dot{\eta}(s)ds \le \varsigma^T(t)\begin{bmatrix} M_1^T + M_1 & -M_1^T + M_2 \\ * & -M_2^T - M_1 \end{bmatrix}\varsigma(t) + \tau(t)\varsigma^T(t)\begin{bmatrix} M_1^T \\ M_2^T \end{bmatrix} S^{-1}\left[M_1, M_2\right]\varsigma(t), \tag{11}$$

where $M_1, M_2 \in \mathbb{R}^{2d}$, $S = S^T > 0$, $\varsigma(t) = [\eta^T(t), \eta^T(t-\tau(t))]^T$, superscript $*$ is an item derived from symmetry $-M_1 + M_2^T$.

Let $\bar{\lambda}_1 = \min\{\lambda^i_{\sigma(t)}\}$, $\bar{\lambda}_2 = \max\{\lambda^i_{\sigma(t)}\}$ $(i \in O_F, \sigma(t) \in \{1, 2, \dots, p\})$, and $\lambda^i_{\sigma(t)}$ be the eigenvalue of real symmetric positive definite matrix $H_{\sigma(t)}$.

Lemma 4. *For any $i \in O_F$, $\sigma(t) \in \{1, 2, \dots, p\}$, $\Theta^i_{\sigma(t)} = \Phi_0 + \lambda^i_{\sigma(t)}\Phi_1 < 0$ if and only if $\Theta_i = \Phi_0 + \bar{\lambda}_i\Phi_1 < 0$ $(i \in \{1, 2\})$.*

Proof. For all $\lambda^i_{\sigma(t)}$, there exists $\beta^i_{\sigma(t)} \in [0, 1]$ satisfying $\lambda^i_{\sigma(t)} = \beta^i_{\sigma(t)}\bar{\lambda}_1 + (1 - \beta^i_{\sigma(t)})\bar{\lambda}_2$, let $\Theta_1 = \Phi_0 + \bar{\lambda}_1\Phi_1$ and $\Theta_2 = \Phi_0 + \bar{\lambda}_2\Phi_1$. Then, $\Theta^i_{\sigma(t)} = \beta^i_{\sigma(t)}\Theta_1 + (1 - \beta^i_{\sigma(t)})\Theta_2$. If $\Theta_i < 0$, one has $\Theta^i_{\sigma(t)} < 0$. Because $\bar{\lambda}_i \in \{\lambda^i_{\sigma(t)}\}$, $\Theta^i_{\sigma(t)} < 0$ denotes $\Theta_i < 0$. This completes the proof. □

The following algorithm presents a strategy to design the protocol and the observers.

Algorithm 1. *For MAS (1) to achieve the state containment, K_1 and K_2 can be determined in these steps.*

Step 1: *Choose suitable K_1 to assign the eigenvalue of $A + BK_1$ at the left-half complex plan. If $A + BK_1$ is controllable, we always have an appropriate K_1.*

Step 2: *Solve LMI (12). If there exist positive symmetric matrices $R = R^T > 0$, $\Omega = \Omega^T > 0$, $S = S^T > 0$, and real matrix $\bar{K}_2$ for any $\bar{\lambda}_i (i = 1, 2)$, LMI (12) is feasible, the gain matric $K_2 = \bar{K}_2\Omega^{-1}$; otherwise, the algorithm stops:*

$$\prod(\bar{\lambda}_i) = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & 0 & R \\ * & \Xi_{22} & \Xi_{23} & \sigma S & 0 \\ * & * & -\sigma S & 0 & 0 \\ * & * & * & -\sigma S & 0 \\ * & * & * & * & -\Omega \end{bmatrix} < 0, \tag{12}$$

where

$$\Xi_{11} = R\tilde{A}^T + \tilde{A}R - \bar{\lambda}_i\bar{K}_2 - \bar{\lambda}_i\bar{K}_2^T - (1-\delta)\Omega$$
$$\Xi_{12} = R - \bar{\lambda}_i\bar{K}_2 - (2-\delta)\Omega$$
$$\Xi_{13} = \sigma R\tilde{A}^T - \sigma\bar{\lambda}_i\bar{K}_2^T$$
$$\Xi_{22} = -(3-\delta)\Omega$$
$$\Xi_{23} = -\sigma\bar{\lambda}_i\bar{K}_2^T.$$

Based on Algorithm 1, the following theorem can be proposed.

Theorem 1. *Using protocol (3) and state observer (4) designed by Algorithm 1, general-linear MAS described by (1) with time-varying delays and switching topologies can achieve the state containment.*

Proof. In order to prove the convergence of the observer, the common Lyapunov-Krasovskii candidate function is constructed as follows:

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{13}$$

where

$$V_1(t) = \varsigma^T(t)\left(I_N \otimes R^{-1}\right)\varsigma(t),$$
$$V_2(t) = \int_{t-\tau(t)}^{t} \varsigma^T(s)\left(I_N \otimes \Omega^{-1}\right)\varsigma(s)ds,$$
$$V_3(t) = \int_{-\sigma}^{0}\int_{t+\mu}^{t} \dot{\varsigma}^T(s)\left(I_N \otimes S^{-1}\right)\dot{\varsigma}(s)dsd\mu.$$

Let $\Lambda_{\sigma(t)} = \mathrm{diag}(\lambda^1_{\sigma(t)}, \lambda^2_{\sigma(t)}, \dots, \lambda^N_{\sigma(t)})$, because $H_{\sigma(t)}$ is positive define and symmetric, it is feasible to find an orthogonal matrix $U_{\sigma(t)}$ that satisfy $U^T_{\sigma(t)}H_{\sigma(t)}U_{\sigma(t)} = \Lambda_{\sigma(t)}$. Let $\eta(t) = (U^T_{\sigma(t)} \otimes I_{Mn})\varsigma(t) = [\eta_1^T(t), \eta_2^T(t), \dots, \eta_N^T(t)]^T$, and $\hat{\eta}_i(t) = [\eta_i^T(t), \eta_i^T(t-\tau(t))]^T$. Because $\varsigma(t)$ is continuously differentiable, so is $V(t)$. It follows that

$$\begin{aligned}
\dot{V}_1(t) &= \dot{\eta}^T(t)(I_N \otimes R^{-1})\eta(t) + \eta^T(t)(I_N \otimes R^{-1})\dot{\eta}(t) \\
&= \eta^T(t)\left(I_N \otimes (R^{-1}\tilde{A} + \tilde{A}^T R^{-1})\right)\eta(t) + 2\eta^T(t)(-\Lambda_{\sigma(t)} \otimes R^{-1}K_2)\eta(t-\tau(t)) \\
&= \sum_{i=1}^{N}\eta_i^T(t)(R^{-1}\tilde{A} + \tilde{A}^T R^{-1})\eta_i(t) + 2\sum_{i=1}^{N}\eta_i^T(t)\left(-\lambda^i_{\sigma(t)}R^{-1}K_2\right)\eta_i(t-\tau(t)) \\
&= \sum_{i=1}^{N}\hat{\eta}_i^T(t)\begin{bmatrix} R^{-1}\tilde{A} + \tilde{A}^T R^{-1} & -\lambda^i_{\sigma(t)}R^{-1}K_2 \\ * & 0 \end{bmatrix}\hat{\eta}_i(t)
\end{aligned} \tag{14}$$

$$\dot{V}_2(t) = \eta^T(t)(I_N \otimes \Omega^{-1})\eta(t) - (1-\dot{\tau}(t))\,\eta^T(t-\tau(t))\,(I_N \otimes \Omega^{-1})\eta(t-\tau(t)). \tag{15}$$

Based on Assumption 3, it follows that

$$\begin{aligned}
\dot{V}_2(t) &\le \eta^T(t)(I_N \otimes \Omega^{-1})\eta(t) - (1-\delta)\eta^T(t-\tau(t))\,(I_N \otimes \Omega^{-1})\eta(t-\tau(t)) \\
&= \sum_{i=1}^{N}\eta_i^T(t)\Omega^{-1}\eta_i(t) - (1-\delta)\sum_{i=1}^{N}\eta_i^T(t-\tau(t))\,\Omega^{-1}\eta_i(t-\tau(t)) \\
&= \sum_{i=1}^{N}\hat{\eta}_i^T(t)\begin{bmatrix} \Omega^{-1} & 0 \\ 0 & -(1-\delta)\Omega^{-1} \end{bmatrix}\hat{\eta}_i(t).
\end{aligned} \tag{16}$$

Let $\Upsilon_i = [\tilde{A}, -\lambda^i_{\sigma(t)}K_2]$, then the time derivative of $V_3(t)$ is shown as

$$\dot{V}_3(t) = \sigma\dot{\eta}^T(t)(I_N \otimes S^{-1})\dot{\eta}(t)\int_{t-\sigma}^{t}\dot{\eta}^T(s)(I_N \otimes S^{-1})\dot{\eta}(s)ds, \tag{17}$$

where

$$\dot{\eta}^T(t)(I_N \otimes S^{-1})\dot{\eta}(t) = \eta^T(t)(I_N \otimes \tilde{A}S^{-1}\tilde{A})\eta(t) - 2\eta^T(t)(\Lambda_{\sigma(t)} \otimes \tilde{A}^T S^{-1}K_2)\eta(t-\tau(t)) + \eta^T(t-\tau(t))\left(\Lambda^2_{\sigma(t)} \otimes K_2^T S^{-1}K_2\right)\eta(t) = \sum_{i=1}^{N}\hat{\eta}_i^T(t)\Upsilon_i^T S^{-1}\Upsilon_i\hat{\eta}_i(t). \tag{18}$$

From Assumption 3 and Lemma 3, one can obtain

$$\begin{aligned}
-\int_{t-\sigma}^{t}\dot{\eta}^T(s)(I_N \otimes S^{-1})\dot{\eta}(s)ds &\leq -\int_{t-\tau(t)}^{t}\dot{\eta}^T(s)(I_N \otimes S^{-1})\dot{\eta}(s)ds \\
&= -\int_{t-\tau(t)}^{t}\sum_{i=1}^{N}\eta_i^T(t)S^{-1}\eta_i(t)ds \\
&= \sum_{i=1}^{N}\left(-\int_{t-\tau(t)}^{t}\eta_i^T(t)S^{-1}\eta_i(t)ds\right) \\
&\leq \sum_{i=1}^{N}\hat{\eta}_i^T(t)\left(\begin{bmatrix} M_1^T + M_1 & -M_1^T + M_2 \\ * & -M_2^T - M_2 \end{bmatrix} + \sigma\begin{bmatrix} M_1^T \\ M_2^T \end{bmatrix} S\,[M_1, M_2]\right)\hat{\eta}_i(t).
\end{aligned} \tag{19}$$

Choose $M_1 = -R^{-1}, M_2 = \Omega^{-1}$. Then, from (13) to (19), we can obtain that

$$\dot{V}(t) \leqslant \sum_{i=1}^{N}\hat{\eta}_i^T(t)Z_i\hat{\eta}_i(t), \tag{20}$$

where

$$Z_i = T_i + \sigma\Upsilon_i^T S^{-1}\Upsilon_i + \sigma\begin{bmatrix} -R^{-T} \\ \Omega^{-T} \end{bmatrix} S\left[-R^{-1}, \Omega^{-1}\right],$$

$$T_i = \begin{bmatrix} T_{i11} & \Omega^{-1} + R^{-1} - \lambda^i_{\sigma(t)}R^{-1}K_2 \\ * & -(3-\delta)\Omega^{-1} \end{bmatrix},$$

$$T_{i11} = -2R^{-1} + R^{-1}\tilde{A} + \tilde{A}^T R^{-1} + \Omega^{-1}.$$

Using Schur complement,[32] $Z_i < 0$ is equal to $\Sigma_i < 0 (i \in O_F)$, where

$$\Sigma_i = \begin{bmatrix} T_i & \sigma\Upsilon_i^T & \sigma[-R^{-1}, \Omega^{-1}]^T \\ * & -\sigma S & 0 \\ * & * & -\sigma S^{-1} \end{bmatrix} < 0.$$

Let $T = \begin{bmatrix} R & 0 \\ \Omega & \Omega \end{bmatrix}$, $\bar{T} = \text{diag}\,\{T, I, S\}$. It can be obtained that

$$\bar{T}^T\Sigma_i\bar{T} = \begin{bmatrix} T^T T_i T & \sigma T^T\Upsilon_i^T & \sigma[0, S]^T \\ * & -\sigma S & 0 \\ * & * & -\sigma S \end{bmatrix}.$$

Because $K_2 = \bar{K}_2\Omega^{-1}, \Pi(\bar{\lambda}_i) < 0 \quad (i = 1, 2)$. Using Schur complement, it can be shown that $\bar{T}^T\Sigma_i\bar{T} < 0$ are equivalent to $\Pi(\lambda^i_{\sigma(t)}) < 0\,(\sigma(t) = 1, 2, \ldots, p, i \in O_F)$. From Lemma 4, if $\Pi(\bar{\lambda}_i) < 0 \quad (i = 1, 2)$, which means $\Pi(\lambda^i_{\sigma(t)}) < 0$, we could propose $Z_i < 0$, one has

$$\lim_{t\to\infty}\varsigma(t) = 0. \tag{21}$$

Recall that $\varsigma(t) = [\tilde{\xi}_1^T(t), \tilde{\xi}_2^T(t), \ldots, \tilde{\xi}_N^T(t)]^T$ and $\tilde{\xi}_i(t) = \hat{\xi}_i(t) - z(t)$. Then, we could propose that $\lim_{t\to\infty}\varsigma(t) = 0$ is equal to $\lim_{t\to\infty}\tilde{\xi}_{i,j}(t) = 0$. The above analyses demonstrate the convergence of the observer.

Note that the containment error is $\dot{\tilde{x}}_i(t) = (A + BK_1)\tilde{x}_i(t) - BK_1 \sum_{j=1}^{M} \rho_{ij}\tilde{\xi}_{i,j}(t)$. Under Algorithm 1, K_1 is designed to satisfying $A + BK_1$ is Hurwitz. We can obtain appropriate K_2 satisfying $\lim_{t\to\infty}\tilde{\xi}_{i,j}(t) = 0$ under Algorithm 1; then, we get $\lim_{t\to\infty}\tilde{x}_i(t) = 0$, which means MAS achieves state containment control with time-varying delays and switching interaction topologies. □

Remark 1. From Theorem 1 and Algorithm 1, one sees that general-linear MAS (1) will achieve state containment with varying time delays and switching interaction topologies. Choosing appropriate K_1, the stability of the protocol can be guaranteed. By solving LMI, the gain matrix K_2 can be specified to ensure the convergence of the state observer.

Remark 2. In the works of Liu and Xie[28] and Wang et al,[29] second-order MASs are studied and there are many restrictions on the sampling time and the maximum delay time. Therefore, it is difficult to use the protocol for general-linear MAS. It should be pointed that by using containment protocol (3), state observer (4), and Algorithm 1, containment control problems for MASs with time-varying delays and switching interaction topologies can be solved directly, which means that the proposed approach in this paper is more versatile. Unlike formation-tracking problem, the target of followers is not to form the formation but to converge to the convex hull spanned by the leaders.

4 | NUMERICAL SIMULATION

In this section, a numerical simulation is proposed to illustrate the effectiveness of the theoretical results.

Choose a third-order MAS with three followers and four leaders, where the dynamic of the system can be described by (1) with $x_i(t) = [x_{i1}(t), x_{i2}(t), x_{i3}(t)]^T i \in \{1, 2, 3\}$, $z_i(t) = [z_{i1}(t), z_{i2}(t), z_{i3}(t)]^T i \in \{1, 2, 3, 4\}$, and $A = \begin{bmatrix} -3 & 1 & -3 \\ 0 & 0 & 1 \\ -5 & -1 & -5 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$.

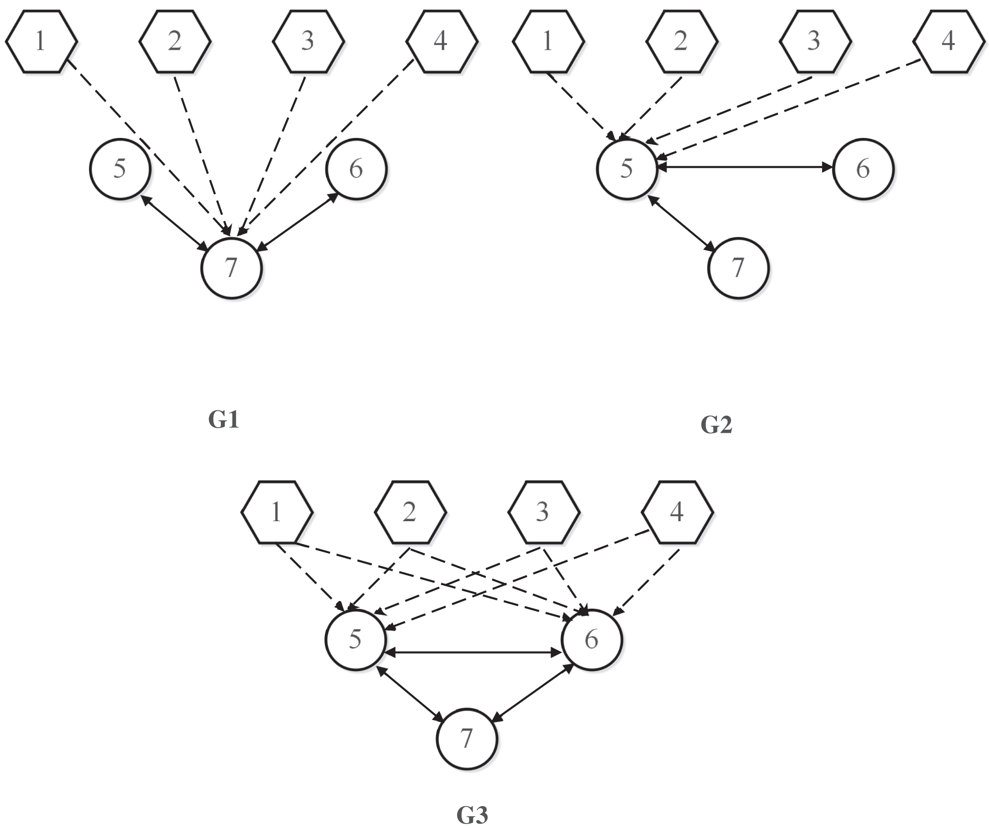

FIGURE 1 Switching interaction topologies

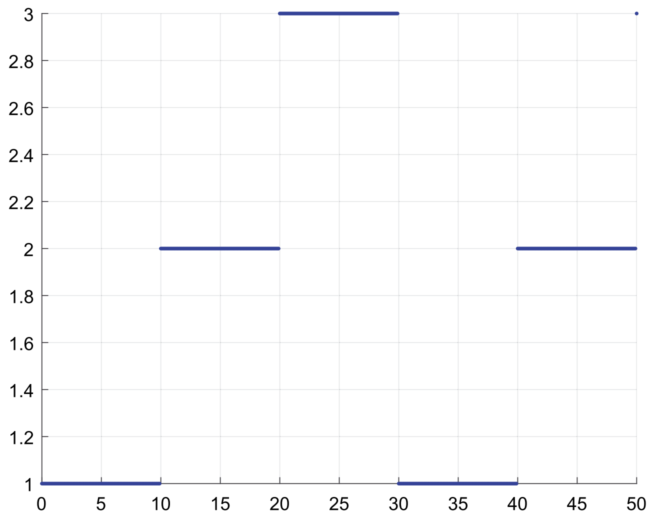

FIGURE 2 Switching signal

The switching interaction topologies are shown in Figure 1. In this example, the interaction topologies are assumed to have 0-1 weights and b_i as well. Suppose that the switching signal $\tau(t) = 0.05 + 0.01\cos(t)$ and the interaction topologies are randomly chosen with $T_d = 10$ seconds. Figure 2 shows the switching signal $\sigma(t)$.

The desired state containment for general linear MAS is specified by $\rho_{51} = \frac{1}{4}$, $\rho_{52} = \frac{1}{4}$, $\rho_{53} = \frac{1}{4}$, $\rho_{54} = \frac{1}{4}$, $\rho_{61} = \frac{2}{3}$, $\rho_{62} = \frac{1}{8}$, $\rho_{63} = \frac{1}{8}$, $\rho_{64} = \frac{1}{12}$, $\rho_{71} = \frac{1}{2}$, $\rho_{72} = \frac{1}{12}$, $\rho_{73} = \frac{1}{12}$, $\rho_{74} = \frac{1}{3}$. The initial state vectors of four leaders are $z_1(0) = [1.2472, -0.2347, -1.0743]^T, z_2(0) = [0.4672, 1.3785, 0.8766]^T$, $z_3(0) = [1.3020, 1.0474, \text{-}\ 1.3929]^T, z_4(0) = [0.7294, 0.7732, 0.5362]^T$. The initial states of three followers are chosen by random numbers, and the initial value of all observers are zeros.

According to Algorithm 1, choose $K_1 = [1.4918\ \text{-}\ 0.5574\ 1.6230]$ to specify the eigenvalues of $A + BK_1$ at -1 , -2, and -3. Solving the LMI (12), $K2$ is given as

$$K_2 = I_4 \otimes \begin{bmatrix} 0.0838 & 0.0784 & -0.3020 \\ 0.0695 & 0.3989 & 0.0584 \\ -0.1230 & -0.0485 & -0.2810 \end{bmatrix}.$$

Figure 3 shows the curves of estimated errors for three observers, and Figure 4 shows curves of state containment errors. As shown in Figure 3, all the followers can acquire leaders' states. Figure 4 demonstrates that the containment errors under switching interaction topologies and time-varying delays converge to zeros. Figure 5 shows the state snapshots of

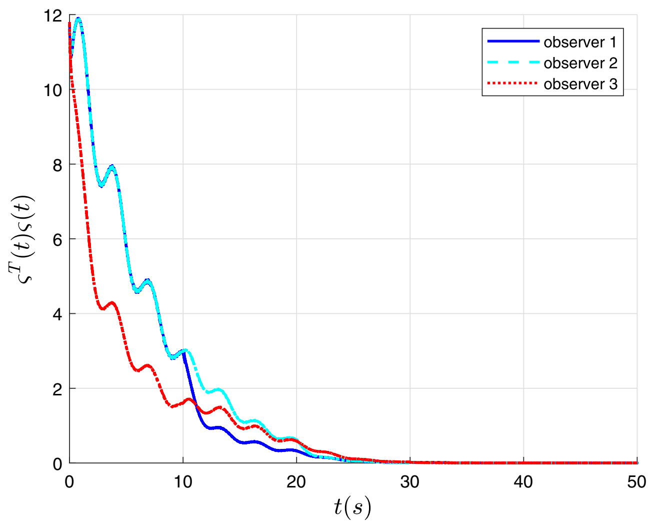

FIGURE 3 Curves of estimation errors

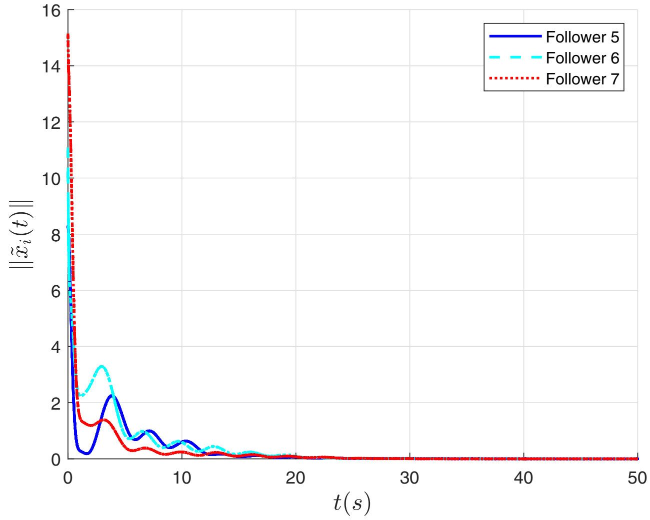

FIGURE 4 Curves of state containment errors

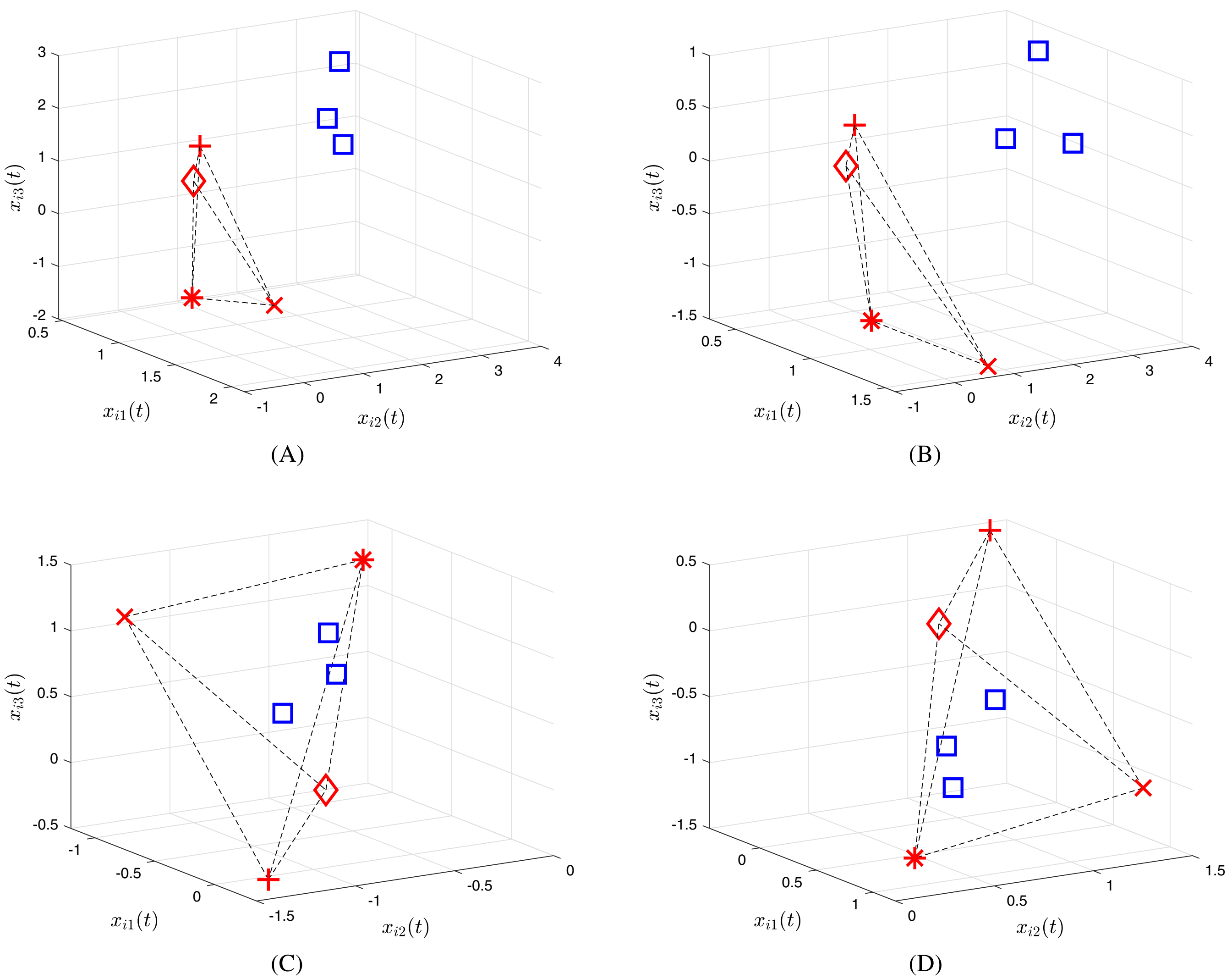

FIGURE 5 Trajectory snapshots of four leaders and three followers at (A) t=0 seconds, (B) t=6 seconds, (C) t=36 seconds, (D) t=50 seconds

six agents, where the states of leaders and followers are indicated by different colors, and the imaginary line is used to represent the convex hull formed by the state of the leaders. It is shown that the states of followers converge to the convex hull spanned by the four leaders. Thus, general linear MAS is said to realize the state containment with time-varying delays and switching interaction topologies.

5 | CONCLUSIONS

Containment control problems for general-linear MASs with switching interaction topologies and time-varying delays were studied. A state observer was constructed under the influence of switching interaction topologies and time-varying delays. Then, a distributed containment protocol was presented, where the expected convex could be specified by several given weights. An approach to specify the gain matrices in the containment protocol and state observer was presented. Using LMI technique and common Lyapunov-Krasovskii stability theory, the convergence of the proposed state observer and the control protocol for general-linear MASs with switching topologies and time-varying delays to realize state containment was proved. The theoretical results were demonstrated by a numerical simulation with four leaders and three followers. A future research direction is to extend the results in this paper to the case where the switching interaction topologies among followers can be directed. Another interesting topic for future study is to consider formation-containment control problems with both time-varying delays and switching interaction topologies.

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ORCID

Shiyu Zhou https://orcid.org/0000-0003-3742-4264
Yongzhao Hua https://orcid.org/0000-0003-1449-0795
Xiwang Dong https://orcid.org/0000-0002-4778-248X

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