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Collision avoidance time-varying group formation tracking control for multi-agent systems

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Abstract
This study considers the time-varying group formation (TVGF) tracking control problem for general linear multi-agent systems (MASs) with collision avoidance, where the MAS is divided into multiple subgroups, enabling followers to form prescribed formations and track trajectories provided by their respective leaders without collisions. Firstly, a distributed TVGF tracking control protocol is introduced using only relative information among neighboring agents. Then, feasibility conditions under which MASs can successfully realize the TVGF tracking without collisions are put forward. Utilizing Lyapunov stability theory, the convergence of the TVGF tracking error systems is confirmed, ensuring the collision-free achievement of the desired formation. Finally, some simulation examples are provided to validate the effectiveness of the theoretical results.

Keywords Distributed control · Networked agent systems · Time-varying formation · Collision free · Stability analysis · Linear dynamics

1 Introduction

Cooperative control of multi-agent systems (MASs) has gained considerable attention over the past decade from diverse fields, such as synchronization of networked agent systems [1, 2], multi-UAV cooperative formation [3], smart transport [4], smart grid [5], etc. Formation control, a typical class of cooperative control that aims at guiding MASs to form a desired formation, has been extensively studied, [6–9] to name a few. Formation control techniques, as outlined in [10], are primarily categorized into virtual structure, behavioral-based, leader-follower, and consensus-based methodologies, with the consensus-based approach, highlighted for its enhanced robustness and scalability, standing out among the others [11].

Reflecting this trend, the consensus-based formation control approach, which requires only neighboring communication among agents to achieve the desired formation, has been widely investigated (see, for example, [11–15]). In [11], a consensus-based formation control scheme was developed for second-order MASs, with its effectiveness demonstrated through a simulation example involving air vehicle formation flying. Dong et al. in [12] designed a formation control protocol for high-order MASs with time delays. It was shown that the formation vectors could be described as time-varying functions, thereby broadening the application scope of formation control. In [13], the time-varying formation control problem with switching topologies was investigated, with the subsequent validation of these theoretical findings through unmanned aerial vehicle (UAV) experiments as detailed in [14]. Wang et al. in [15] introduced a fully adaptive dis-

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tributed time-varying formation controller for high-order MASs without requiring global information.

The above-mentioned works are primarily concentrated on the leaderless formation problem. However, in some practical applications, such as collaborative fencing, the agents are required to not only form a specific time-varying formation but also track a specific trajectory simultaneously. Therefore, the so-called time-varying formation tracking (TVFT) control problem has been widely investigated [16–19]. In [17], the authors designed a TVFT control protocol for MASs with directed topology, establishing the feasibility conditions for the desired formation. To address complex tasks, the assignment of multiple leaders may be essential. The authors in [18] proposed the TVFT control scheme for MASs with multiple leaders, where the followers need to not only shape predefined time-varying formation but also achieve tracking for convex combinations of multiple leaders' states. A sampled-data-based TVFT control scheme was designed for MASs with multiple leaders, where the sampling interval can be relatively chosen to be large, thus reducing congestion of network bandwidth resources [19].

Actually, in complex applications such as multi-target enclosing, cooperative area search, etc., multiple agents need to be divided into multiple subgroups to perform different tasks. However, the above literature considered all the agents belonging to one group and cannot be applied to solve multi-task collaboration. As such, group coordination problems of MASs have been a focus of numerous studies, covering various topics such as group consensus, group formation, and group formation tracking, see, e.g., [20–24]. In [20], the group consensus problem for MASs with both fixed and switching topologies was studied. The time-varying group formation (TVGF) problem for general linear MASs was investigated in [21], and the feasibility conditions for achieving group formation were also provided. The extension works with respect to the adaptive TVGF tracking control scheme were proposed in [22] where the global Laplacian matrix information need not be known in advance. In [23], the TVGF tracking problem in the presence of switching networks was investigated, where the trajectory of each group was determined via the command input of its nonautonomous leader. However, it is important to note that these studies reported in [20–24] primarily focus on achieving group formations, without addressing the potential for collisions between agents.

Apart from realizing the desired TVGT tracking, collision avoidance is of great significance in guaranteeing the safety and performance of large-scale MASs. The commonly used strategies for collision avoidance can be categorized into artificial potential field (APF) methods and geometric guidance methods [25]. The APF-based collision-free formation controllers for first-order and second-order MASs were investigated in [26–28]. These studies focus on developing appropriate potential functions to achieve global asymptotic convergence of the concerned formation tracking errors. More recently, collision-free formation-tracking problems have been studied for multi-autonomous underwater vehicle systems [25] and nonlinear MASs [29], respectively, where the position-tracking errors were proven to be bounded. Nevertheless, the aforementioned APF-based methods cannot avoid the problem of local minimum, rendering them incapable of achieving the asymptotic convergence of position tracking errors. This issue is particularly pronounced in complex environments where the potential field's gradient descent can lead to suboptimal paths or even deadlocks near local minima. In addition, based on the geometric guidance approach, a distributed formation control approach with obstacle avoidance was put forward for a single group [30, 31] and multiple groups [32], respectively. However, unfortunately, the geometric guidance approach suffers from one undesirable disadvantage of high computational cost. This is due to the complex calculations involved in determining the optimal path, which can be particularly demanding in real-time applications.

According to the above discussion, it can be observed that the problem of TVGF tracking problem with collision avoidance for MASs has not been well addressed, which motivates this study. This study addresses the TVGF tracking control problem for MASs with the ability of collision avoidance. In this problem, each group of followers can form the desired formation shape and track the different trajectories provided by individual leaders without collision. The main contributions of this work can be summarized as follows:

1) A distributed TVGF tracking control scheme is proposed for linear MASs and feasibility conditions for collision-free among agents are derived. Compared with single-group formation control methods [13, 14, 16–19, 33–35], the TVGF tracking control problem for multiple groups is considered in this study. These subgroups are required to not only form the time-varying formations but also to track their respective leaders. Due to the need for communication both within and between groups, the TVGF tracking problem is more challenging in both controller design and stability analysis.

2) The proposed collision avoidance feasibility conditions in this work eliminate the need for incorporating additional forces, setting it apart from existing APF-based collision-free studies [25–29]. This innovation simplifies the control strategy while still ensuring the convergence of tracking errors, making it an efficient and practical approach. The proposed feasibility conditions ensure that collisions are avoided as long as the established criteria are satisfied, thus resulting in smoother trajectories and improved motion planning predictability.

3) Unlike existing methods [30–32] that rely on geometric guidance and often assume objects move with constant velocities, the presented collision avoidance approach merely

restricts the initial states of MASs. This departure from traditional methods reduces the computational burden and offers a more adaptable solution for real-world applications. Furthermore, different from the works of [25–32], more general agent dynamics is considered in this study.

The rest of this study is organized as follows: Section 2 introduces some preliminaries and formulates the TVGF tracking problem. The distributed controllers with collision avoidance feasibility conditions are given in Section 3. Moreover, the analysis of the resulting closed-loop systems is also presented. In Section 4, a simulation example is given to illustrate the effectiveness of the proposed controllers. Section 5 concludes the paper.

2 Preliminaries and problem formulation

2.1 Notations

In this study, $\mathbf{1}_M$ is the $M \times 1$ vector whose every entry is one. The n dimensional identity matrix is denoted as I_n. $\otimes$ represents the Kronecker product. The diagonal block matrix is defined as $\mathrm{diag}\{d_1, \ldots, d_i, \ldots, d_N\}$ with d_i as its diagonal entry. $\|\cdot\|$ represents the Euclidean norm. $\min(\cdot)$ and $\max(\cdot)$ return the minimum and maximum elements of an array, respectively. For a matrix $\mathcal{M} \in \mathbb{R}^{n\times n}$ with all the eigenvalues being real, $\lambda_{\min}(\mathcal{M})$ and $\lambda_{\min}(\mathcal{M})$ represent the maximum and minimum eigenvalues of $\mathcal{M}$, respectively. $\exp(\cdot)$ denotes the exponential function.

2.2 Problem statement

Consider a MAS with M followers and N leaders. The system is partitioned into N subgroups denoted as $\mathcal{O}_k, k = 1, \ldots, N$, each consisting of one leader and g_i followers, with $\sum_{i=1}^{N} g_i = M$. Let $\mathcal{O}_f = \{1, 2, \ldots, M\}$ and $\mathcal{O}_l = \{1, 2, \ldots, N\}$ denote the set of followers and leaders, respectively. The partition of the follower set is defined as $\{\mathcal{O}_{f1}, \mathcal{O}_{f2}, \ldots, \mathcal{O}_{fN}\}$ satisfying $\mathcal{O}_{fj} \neq \varnothing\,(j = 1, 2, \ldots, N)$, $\cup_{j=1}^{N}\mathcal{O}_{fj} = \mathcal{O}_f$, and $\mathcal{O}_{fj} \cap \mathcal{O}_{fs} = \varnothing$ $(j, s \in \{1, 2, \ldots, N\};\ j \neq s)$. It can be seen that $\mathcal{O}_k$ consists of the k-th leader and the subset of followers $\mathcal{O}_{fk}$.

The dynamic of the i-th $\left(i \in \mathcal{O}_f\right)$ follower is given as:

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t),\ i = 1, 2, \ldots, M, \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$, $u_i(t) \in \mathbb{R}^m$ denote the state, control input, and output of the i-th follower, respectively, $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$ are the system matrices satisfying $\mathrm{rank}(B) = m$.

The dynamic of the leader is given as

$$\dot{x}_{0j}(t) = Ax_{0j}(t),\ j = 1, 2, \ldots, N, \tag{2}$$

where $x_{0j}(t) \in \mathbb{R}^n$ represent the state of the leader.

This study considers the problem of TVGF tracking with collision avoidance. For any subgroup $\mathcal{O}_k = \{\text{leader } k, \mathcal{O}_{fk}\}$ $(k = 1, 2, \ldots, N)$, let $h_{xi}(t) \in \mathbb{R}^n$ represent the piecewise continuous differentiable time-varying formation vector. We make the following assumptions on the collision avoidance and time-varying formation vector.

Assumption 1 Appropriate reference trajectories for the leader agents are pre-given, ensuring that potential collisions only occur among the followers within the same subgroup.

Providing pre-defined reference trajectories for leader agents simplifies the system's complexity, enhancing the followers' ability to adapt to the leader's motion planning and mitigating collision risks. This approach ensures clear motion guidance, avoiding collisions among leaders and facilitating followers' path tracking, thereby reducing the coordination complexity between leaders and followers. Additionally, confining potential collisions to within subgroups of followers simplifies collision avoidance. By restricting the scope of potential collisions, the need to consider collision avoidance scenarios is reduced, lowering the complexity of the entire system and improving the efficiency of motion planning and collision avoidance.

Assumption 2 For any subgroup $\mathcal{O}_k = \{\text{leader } k, \mathcal{O}_{fk}\}, k = 1, 2, \ldots, N$,

$$\min(\|h_i(t) - h_j(t)\|) =: \gamma_k > r_{\min} > 0, \tag{3}$$

where $i, j \in \mathcal{O}_{fk}$, $i \neq j$, $r_{\min}$ represents the minimum distance required between the followers to ensure collision avoidance.

Remark 1 Assumption 1 ensures that there is no collision between different groups. Assumption 2 represents that when MASs achieve the TVGF tracking control, the separation distance between the followers in the same group will be greater than the required minimum distance $r_{\min}$.

Then, based on Assumptions 1 and 2, the TVGF tracking problem with collision avoidance can be described as follows.

Definition 1 (TVGF tracking problem with collision avoidance) Given the MAS consisting of (1) and (2), the TVGF tracking problem with collision avoidance is said to be solved if there exists a distributed protocol for each follower in any subgroup $\mathcal{O}_k = \{\text{leader } k, \mathcal{O}_{fk}\}, k = 1, 2, \ldots, N$, such that

$$i)\ \lim_{t\to\infty}(x_i(t) - h_i(t) - x_{0k}(t)) = 0,\ \forall i \in \mathcal{O}_{fk}, \tag{4}$$

$$ii)\ \left\|x_i(t) - x_j(t)\right\| > r_{\min},\ \forall t \geq 0,\ i, j \in \mathcal{O}_{fk},\ i \neq j. \tag{5}$$

Remark 2 Equation (4) indicates the asymptotic convergence of the TVGF tracking errors, implying that the followers in each group will achieve the desired formation and track the trajectory provided by the corresponding leader. Meanwhile, (5) demonstrates that no collision occurs among the followers in each group.

3 Main results

In this section, the main results of this article will be presented. To proceed, the following assumptions are made.

Assumption 3 For any $i \in \mathcal{O}_f$, the matrices pair (A, B) are stabilizable.

Assumption 4 The partition $\{\mathcal{O}_{f1}, \mathcal{O}_{f2}, \ldots, \mathcal{O}_{fN}\}$ is an acyclic partition for the set of followers $\mathcal{O}_f$.

Assumption 5 For each subgroup $\mathcal{O}_k = \{\text{leader } k, \mathcal{O}_{fk}\}$, $k = 1, 2, \ldots, N$, the communication topology among the followers in the subgroup contains a spanning tree with the corresponding leader serving as its root.

Let $\mathcal{G}_F = (\mathcal{O}_f, \mathcal{E}_f, \mathcal{W}_f)$ represents the digraph among followers, which consist of the set of nodes $\mathcal{O}_f$, the set of edges $\mathcal{E}_f$, and the weighted adjacency matrix $\mathcal{W}_f = [w_{ij}]_{M\times M}$, where $w_{ij} > 0$ if $(j, i) \in \mathcal{E}_f$ and otherwise, $w_{ij} = 0$. The pinning gains from the k-th leader to each follower i are represented by w_{i0k}, where $w_{i0k} > 0$ if the information can be transmitted from the leader k to the follower i; otherwise, $w_{i0k} = 0$. The Laplacian matrix of the MAS can be represented as follows:

$$\mathcal{L}_{FL} = \begin{bmatrix} \mathcal{L}_1 & \mathcal{L}_2 \\ 0_{N\times M} & 0_{N\times N} \end{bmatrix},$$

where $\mathcal{L}_2 = [-w_{0ik}]_{M\times N}$, $\mathcal{L}_1 = [l_{ij}]_{M\times M}$ with $l_{ij} = -w_{ij}$ for $i \neq j$, and $l_{ij} = \sum_{m=1}^{M} w_{im} + \sum_{k=1}^{N} w_{i0k}$ for $i = j$.

It follows from Assumptions 3 and 4 that $\mathcal{L}_1$ has the following form [23]

$$\mathcal{L}_1 = \begin{bmatrix} \mathcal{L}_{f_1} & 0 & \cdots & 0 \\ \mathcal{L}_{f_{21}} & \ddots & & 0 \\ \vdots & \mathcal{L}_{f_{ij}} & \ddots & \vdots \\ \mathcal{L}_{f_{N1}} & \cdots & \cdots & \mathcal{L}_{f_N} \end{bmatrix},$$

where $\mathcal{L}_{fi}$ represents the interaction among the followers in subgroup $\mathcal{O}_{fi}$. The interaction between the followers of subgroups $\mathcal{O}_{fi}$ and $\mathcal{O}_{fj}$ is defined as $\mathcal{L}_{f_{ij}}$.

Assumption 6 For any given subgroups $\mathcal{O}_i$ and $\mathcal{O}_j$, $i, j \in \{1, 2, \ldots, N\}$, $i \neq j$, the sum of each row in $\mathcal{L}_{fij}$ is equal to zero.

We summarize some important properties of $\mathcal{L}_1$ and $\mathcal{L}_2$ as follows.

Lemma 1 ([36]) *Under Assumptions 4-6, all eigenvalues of $\mathcal{L}_1$ have positive real parts.*

Lemma 2 ([23]) *Under Assumptions 4-6, there exists a real diagonal matrix $\mathcal{D} = \mathrm{diag}\{d_1, d_2, \ldots, d_M\}$, such that $\mathcal{D}\mathcal{L}_1 + \mathcal{L}_1^T\mathcal{D}$ is positive definite, where $d_i > 0$, $i = 1, 2, \ldots, M$.*

Lemma 3 ([22]) *Under Assumptions 4-6, $-\mathcal{L}_1^{\mathrm{T}}\mathcal{L}_2$ has the following form*

$$-\mathcal{L}_1^{\mathrm{T}}\mathcal{L}_2 = \begin{bmatrix} \bar{w}_1 & 0 & \cdots & 0 \\ 0 & \bar{w}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \bar{w}_N \end{bmatrix},$$

where $\bar{w}_i = \mathbf{1}_{g_i}$, $i = 1, 2, \ldots, N$.

In the following, we present a TVGF tracking protocol and the collision avoidance feasibility conditions for each group.

For any subgroup $\mathcal{O}_k = \{\text{leader } k, \mathcal{O}_{fk}\}, k = 1, 2, \ldots, N$, the TVGF tracking protocol is constructed as follows:

$$u_i(t) = K\delta_i(t) + r_i(t),\ \ i \in \mathcal{O}_{fk}, \tag{6}$$

where,

$$\begin{aligned} \delta_i(t) = & \sum\nolimits_{j=1}^{M} w_{ij}((x_i(t) - h_i(t)) - (x_j(t) - h_j(t))) \\ & + \sum\nolimits_{k=1}^{N} w_{i0k}(x_i(t) - h_i(t) - x_{0k}(t)), \end{aligned}$$

K is the gain matrix and $r_i(t)$ is the formation compensation item that can determined later. According to this multi-leader-multi-follower organization structure, multiple leaders can spearhead different subgroups to conduct different tasks. Considering the presence of multiple leaders makes it challenging to model and analyze. The distributed cooperative control scheme designed in this study can be applied to tasks such as multi-target enclosing and decentralized cooperative area monitoring.

Since rank $B = m$, there exists a nonsingular matrix $T = [\bar{B}^{\mathrm{T}}, \hat{B}^{\mathrm{T}}]^{\mathrm{T}}$ that satisfies the conditions $\bar{B}B = \mathrm{I}_m$, $\hat{B}B = 0$, $\bar{B} \in \mathbb{R}^{m\times n}$, $\hat{B} \in \mathbb{R}^{(n-m)\times n}$, and $n > m$. The parameters in the TVGF tracking protocol (20) can be designed through the following Algorithm.

Algorithm 1: The Algorithm to design the TVGF tracking protocol (6) is described as the following three steps:

Step 1: For each group k ($k = \{1, 2, \cdots, N\}$), $i \in \mathcal{O}_{fk}$, calculate the following TVGF tracking feasibility condition

$$\hat{B}(Ah_i(t) - \dot{h}_i(t)) = 0, \tag{7}$$

if this condition is satisfied, the algorithm continues; otherwise, the formation parameter $h_i(t)$ should be redesigned.

Step 2: The compensation item $r_i(t)$ can be calculated using the following equation:

$$r_i(t) = -\bar{B}\left(Ah_i(t) - \dot{h}_i(t)\right). \tag{8}$$

Step 3: Solve the following equation to get the positive definite matrix P

$$PA + A^{\mathrm{T}}P - PBB^{\mathrm{T}}P + \mathrm{I}_n = 0, \tag{9}$$

Then $K = -\gamma B^{\mathrm{T}}P$, where $\gamma = \frac{\lambda_{\max}(D)}{\lambda_{\min}(D\mathcal{L}_1+\mathcal{L}_1^{\mathrm{T}}D)}$.

Now, the effectiveness of the TVGF tracking protocol with collision avoidance is shown in the following Theorem.

Theorem 1 *Suppose that* Assumptions 1-6 *hold. If the TVGF tracking feasibility condition* (7) *is satisfied and the initial tracking error systems for the subgroup* $\mathcal{O}_k = \{\text{leader } k, \mathcal{O}_{fk}\}$, $k = 1, 2, \ldots, N$, *satisfy*

$$\|\bar{e}_k(0)\| < \frac{1}{k_e\sqrt{g_k}}\left(\gamma_k - r_{\min}\right), \tag{10}$$

where $\bar{e}_k(0) = [e^{\mathrm{T}}_{\Xi_k+1}(t), \ldots, e^{\mathrm{T}}_{\Xi_k+g_k}(t)]^{\mathrm{T}}$ *with*

$$\Xi_k = \sum\nolimits_{j=1}^{k-1} g_j,\ k_e = \sqrt{\frac{\lambda_{\max}(D\mathcal{L}_1 + \mathcal{L}_1^{\mathrm{T}}D)\lambda_{\max}(P)}{\lambda_{\min}(D\mathcal{L}_1 + \mathcal{L}_1^{\mathrm{T}}D)\lambda_{\min}(P)}},$$

the TVGF tracking problem with collision avoidance described by Definition 1 *is addressed under the protocol* (6) *designed by Algorithm 1.*

Based on Algorithm 1 and Theorem 1, the control flowchart for achieving the collision avoidance time-varying group formation tracking is shown in Fig. 1. Specifically, the control scheme consists of the following key steps: 1) Construct the multi-agent system and judge the achievability of obstacle avoidance; 2) Calculate the time-varying formation tracking condition and ensure the feasibility of formation control; 3) Calculate the formation compensation term to improve the control accuracy; 4) Solve the Algebraic Riccati Equation to obtain the control gain; and 5) Achieve the time-varying group formation tracking control. In the following, a further proof of Theorem 1 is given.

Proof The proof of Theorem 1 consists of two parts: the asymptotic convergent of the TVGF tracking errors, and no collisions between the followers in each group can occur while accomplishing TVGF tracking.

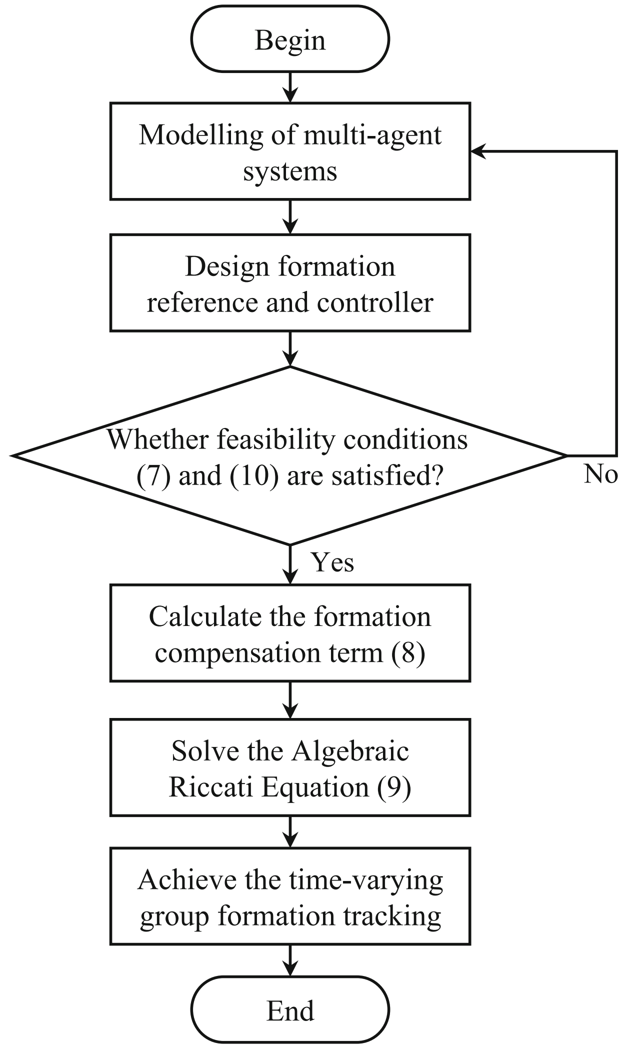

Fig. 1 The control flowchart for achieving the collision avoidance time-varying group formation tracking

i) Let $e_i(t) = x_i(t) - x_{0k}(t) - h_i(t)$. It follows from (6) and (1) that

$$\dot{e}_i(t) = Ae_i(t) - BK\delta_i(t) - Br_i(t) + Ah_i(t) - \dot{h}_i(t). \tag{11}$$

Combining the TVGF tracking feasibility condition (7) and formation compensation item (8), one has

$$-Br_i(t) + Ah_i(t) - \dot{h}_i(t) = 0. \tag{12}$$

Let $e(t) = [e_1^{\mathrm{T}}(t), \ldots, e_M^{\mathrm{T}}(t)]^{\mathrm{T}}$, $\delta(t) = [\delta_1^{\mathrm{T}}(t), \ldots, \delta_M^{\mathrm{T}}(t)]^{\mathrm{T}}$, $x(t) = [x_1^{\mathrm{T}}(t), \ldots, x_M^{\mathrm{T}}(t)]^{\mathrm{T}}$, $h(t) = [h_1^{\mathrm{T}}(t), \ldots, h_M^{\mathrm{T}}(t)]^{\mathrm{T}}$, and $x_0(t) = [x_{01}^{\mathrm{T}}(t), \ldots, x_{0N}^{\mathrm{T}}(t)]^{\mathrm{T}}$.

According to the Lemma 3, one can obtain that $\delta(t)$ is

$$\begin{aligned}\delta(t) =& (\mathcal{L}_1 \otimes \mathrm{I}_n)\left(x(t) - h(t) + (\mathcal{L}_1^{-1}\mathcal{L}_2 \otimes \mathrm{I}_n)x_0(t)\right)\\ =& (\mathcal{L}_1 \otimes \mathrm{I}_n)e(t).\end{aligned}$$

Then, it follows from (11) and (12) that

$$\dot{\delta}(t) = (\mathrm{I}_M \otimes A + \mathcal{L}_1 \otimes BK)\delta(t). \tag{13}$$

Designing the following Lyapunov function candidate

$$V_\delta(t) = \delta^{\mathrm{T}}(t)(D \otimes P)\delta(t). \tag{14}$$

Taking the time derivative of (14), one has

$$\begin{aligned}\dot{V}_\delta(t) &= \delta^{\mathrm{T}}(t)\left(D \otimes (PA + A^{\mathrm{T}}P)\right)\delta(t)\\ &+ \delta^{\mathrm{T}}(t)\left(D\mathcal{L}_1 \otimes PBK + \mathcal{L}_1^{\mathrm{T}} D \otimes K^{\mathrm{T}}B^{\mathrm{T}}P\right)\delta(t).\end{aligned} \tag{15}$$

Note that $K = -\gamma B^{\mathrm{T}}P$ with

$$\gamma = \frac{\lambda_{\max}(D)}{\lambda_{\min}(D\mathcal{L}_1 + \mathcal{L}_1^{\mathrm{T}} D)}.$$

According to the Lemma 2, one can obtain that

$$\begin{aligned}&\delta^{\mathrm{T}}(t)\left(D\mathcal{L}_1 \otimes PBK + \mathcal{L}_1^{\mathrm{T}} D \otimes K^{\mathrm{T}}B^{\mathrm{T}}P\right)\delta(t)\\ &\le -\delta^{\mathrm{T}}(t)\left(D \otimes PBB^{\mathrm{T}}P\right)\delta(t).\end{aligned} \tag{16}$$

It follows from (16) and (9) that

$$\dot{V}_\delta(t) \le -\delta^{\mathrm{T}}(t)\left(D \otimes \mathrm{I}_n\right)\delta(t) \le -\frac{1}{\lambda_{\max}(P)}V_\delta(t) \tag{17}$$

From (17), one can conclude that $\lim_{t\to\infty}\delta(t) = 0$ exponentially. Recalling that $\delta(t) = (\mathcal{L}_1 \otimes \mathrm{I}_n)e(t)$ and $\mathcal{L}_1$ is nonsingular, one has $\lim_{t\to\infty}e(t) = 0$ exponentially, which means TVGF tracking problem described by (4) is addressed.

ii) The following will prove that collisions will not occur. Since $\delta(t) = (\mathcal{L}_1 \otimes \mathrm{I}_n)e(t)$, via (17) and Theorem 4.10 in [37], one can obtain that

$$\|e(t)\| \le k_e\|e(0)\|\exp(-\alpha t) \le k_e\|e(0)\|, \tag{18}$$

where $k_e = \sqrt{\frac{\lambda_{\max}(D\mathcal{L}_1+\mathcal{L}_1^{\mathrm{T}}D)\lambda_{\max}(P)}{\lambda_{\min}(D\mathcal{L}_1+\mathcal{L}_1^{\mathrm{T}}D)\lambda_{\min}(P)}}$, $\alpha = -\frac{1}{2\lambda_{\max}(P)}$.

From (18), one has $\bar{e}_k(t) \le k_e\bar{e}_k(0)$, $k = 1, 2, \ldots, N$. Then, according to Assumptions 1 and 2, one can get

$$\begin{aligned}\|x_i(t) - x_j(t)\| &= \|e_i(t) - e_j(t) + h_i(t) - h_j(t)\|\\ &\ge \|h_i(t) - h_j(t)\| - \|e_i(t)\| - \|e_j(t)\|\\ &\ge \gamma_k - \sum\nolimits_{i=1}^{g_k}\|e_i(t)\|.\end{aligned} \tag{19}$$

According to Hardy's inequality and (18), one has $\sum_{i=1}^{g_k}\|e_i(t)\| \le \sqrt{g_k}\|\bar{e}_k(t)\| \le k_e\sqrt{g_k}\|\bar{e}_k(0)\|$. Then, combining the collision avoidance feasibility condition (10), one has

$$\|x_i(t) - x_j(t)\| \ge \gamma_k - k_e\sqrt{g_k}\|\bar{e}_k(0)\| > r_{\min}. \tag{20}$$

Form (20), one can conclude that collision-free trajectories can be generated for the followers within each group. This completes the proof. □

In this section, the TVGF tracking control scheme was designed for generic linear MASs. The corresponding collision avoidance feasibility conditions were obtained by analyzing the stability of the closed-loop system. Thus, all the followers can be divided into different subgroups according to the requirements, and then not only do the subgroups realize the time-varying formation, but also the followers can track the corresponding leaders with collision avoidance.

Remark 3 In addition to achieving the TVGF tracking described by (4), Theorem 1 also takes into account collision avoidance among the followers within each group. It is important to highlight that (10) serves as a sufficient condition for achieving collision avoidance within each group. By ensuring that the initial states of MASs are configured to satisfy (10), collisions among followers within the group can be effectively prevented. It should be noted that (10) is derived based on the linearity of the system dynamics, which allows us to apply Lyapunov stability theory to confirm the convergence of the TVGF tracking error systems. To extend the collision avoidance feasibility conditions to nonlinear systems, one would need to adapt the control protocol and stability analysis. Specifically, the control laws would likely require redesign to accommodate non-linear dynamics, and the stability analysis might involve more advanced techniques such as nonlinear Lyapunov functions or other robustness analyses.

Remark 4 In this study, a collision avoidance TVGF tracking control scheme is designed for generic linear MASs. Different from the previous APF-based collision-free studies [25–29], the proposed collision avoidance feasibility conditions do not require incorporating extra forces. Consequently, this approach effectively addresses the issue of local minima arising from APF while also ensuring the stability of tracking errors. Moreover, the geometric guidance-based methods used in [30–32] often assume that objects move with constant velocities and require extensive computation. In contrast, our proposed collision avoidance method merely restricts the initial states of MAS. Consequently, this method has the advantages of minimal computational complexity and high potential for its wide application in practice.

Remark 5 Specifically, the obstacle avoidance presented in this study is primarily focused on collision avoidance among agents and does not extend to navigating complex external environments with static/dynamic obstacles. Additionally, we consider homogeneous MASs, which may not fully represent the dynamics of heterogeneous systems. These limitations have indeed inspired our future research directions. We plan to address these issues in subsequent studies by designing obstacle-avoidance and time-varying formation control schemes for heterogeneous MASs, taking into account the complexities of the external environment.

4 Simulation

This section gives simulations to illustrate the effectiveness of the TVGF tracking protocol and the collision avoidance feasibility conditions. Given that MASs consist of 13 agents, they are divided into three groups. The leaders are represented by yellow hexagons and the followers are represented by blue circles. Specifically, the first group is comprised of one leader and three followers, the second group also has one leader and three followers, and the third group comprises one leader and four followers. The communication network is visually depicted in Fig. 2.

4.1 Time-varying group formation tracking for general linear multi-agent systems

In this part, some simulation results are given to verify the effectiveness of the proposed collision avoidance

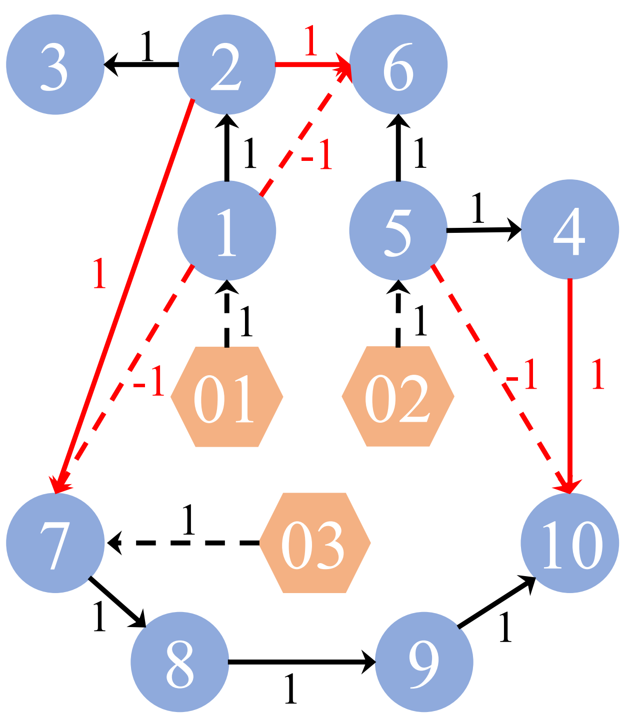

Fig. 2 The interaction topology among agents

TVGF tracking control scheme for general linear MASs. The dynamics of agents are denoted as follows [23]:

$$A = \begin{bmatrix} 0 & 1 & -0.15 \\ 0 & 0 & 1.5 \\ 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

The time-varying formation vectors for each group are denoted as follows:

$$h_i = 15 \begin{pmatrix} \sin\left(t + \frac{(i-1)2\pi}{3}\right) \\ \cos\left(t + \frac{(i-1)2\pi}{3}\right) \\ -\sin\left(t + \frac{(i-1)2\pi}{3}\right) \end{pmatrix}, \quad i = 1, 2, 3,$$

$$h_j = 10 \begin{pmatrix} \sin\left(t + \frac{(j-4)2\pi}{3}\right) \\ \cos\left(t + \frac{(j-4)2\pi}{3}\right) \\ -\sin\left(t + \frac{(j-4)2\pi}{3}\right) \end{pmatrix}, \quad j = 4, 5, 6,$$

$$h_k = 15 \begin{pmatrix} \sin\left(t + \frac{(k-7)\pi}{2}\right) \\ \cos\left(t + \frac{(k-7)\pi}{2}\right) \\ -\sin\left(t + \frac{(k-7)\pi}{2}\right) \end{pmatrix}, \quad k = 7, 8, 9, 10.$$

It can be verified that the TVGF tracking feasibility conditions (7) are satisfied. The compensation items can be calculated by the (8).

According to Theorem 1 and Algorithm 1, the positive definite matrix P and the gain matrix K are:

$$P = \begin{bmatrix} 2.257 & 2.048 & 1 \\ 2.048 & 3.652 & 2.257 \\ 1 & 2.257 & 2.734 \end{bmatrix}, \quad K = \begin{bmatrix} -28 \\ -63.2 \\ -76.5 \end{bmatrix}^T.$$

4.1.1 Verifying the effectiveness of TVGF tracking protocol

Firstly, the effectiveness of the TVGF tracking protocol is verified. As a result, it is not imperative for the conditions described in (10) to be satisfied at the initial simulation time.

The initial states of followers and leaders are given as $x_i(0) = [5\varpi, \varpi, 5\varpi]^{\mathrm{T}}$ and $x_{0k}(0) = [\varpi, \varpi, \varpi]^{\mathrm{T}}$, where ϖ denotes the random number between -0.5 and 0.5.

Figure 3 illustrates the state trajectory of MASs within the simulation time. In this figure, groups 1-3 are respectively indicated by the colors red, green, and blue. Symbols $*$ and $\circ$ are used to represent the leaders and the followers, respectively. The presented numerical simulation validates the effectiveness of the proposed TVGF tracking protocol. Furthermore, in order to show the evolution of the 3D position states of the agents in each subgroup, we selected four moments, i.e., $t = 0s$, $t = 10s$, $t = 20s$, and $t = 30s$. The simulation results are shown in Figs. 4-6. In particular,

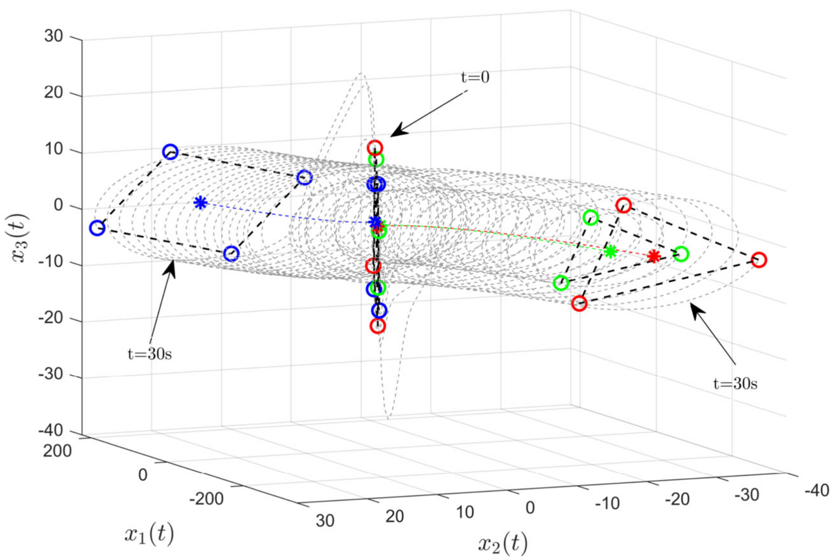

Fig. 3 The state trajectories of agents

the formation shapes formed by all the agents at the current moment are shown in each subgraph.

4.1.2 Verifying the effectiveness of TVGF tracking protocol with collision avoidance feasibility conditions

Moreover, the requirement of collision avoidance should be taken into consideration. The minimum distance between the MASs is given as $r_{\min} = 1.5$. The initial states and observer error systems of the MASs should be reassigned to meet the collision avoidance feasibility conditions (10).

Figure 7 depicts the TVGF tracking errors and the distance between groups within the simulation time. Figure 7 shows that the TVGF tracking errors converge to zeros as time goes to infinity. It can be observed that distances between individuals are greater than $r_{\min}$. Thus, the MASs can realize the TVGF tracking control with collision avoidance.

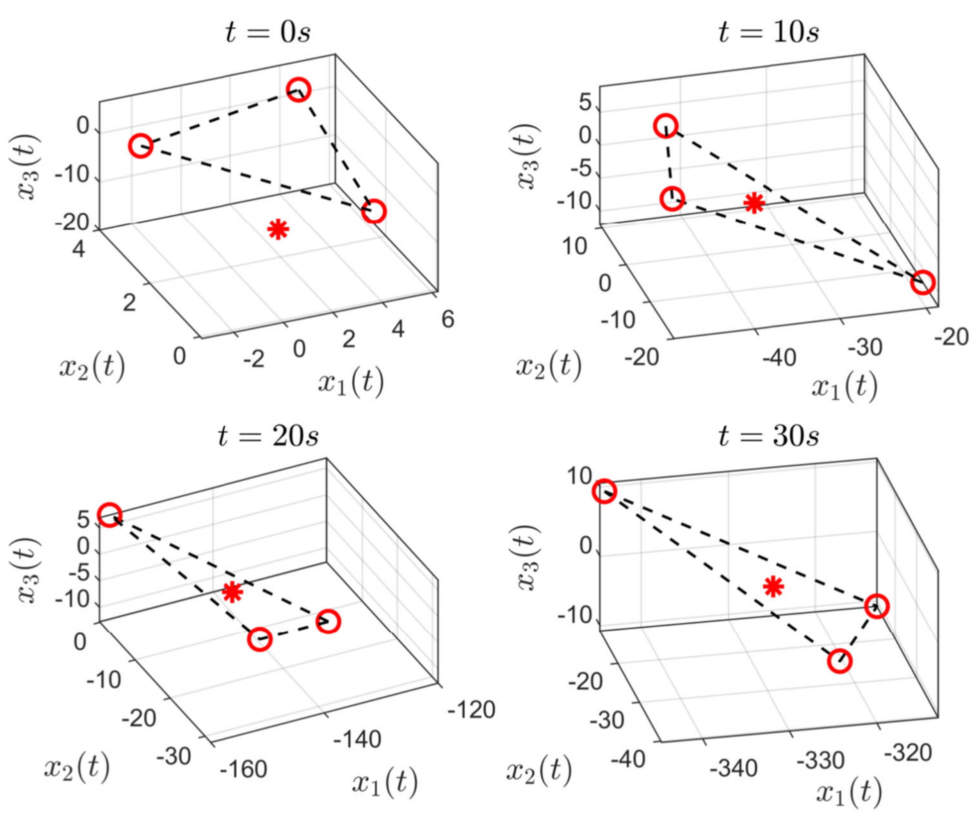

Fig. 4 The position trajectories of agents in subgroup $\mathcal{O}_1$

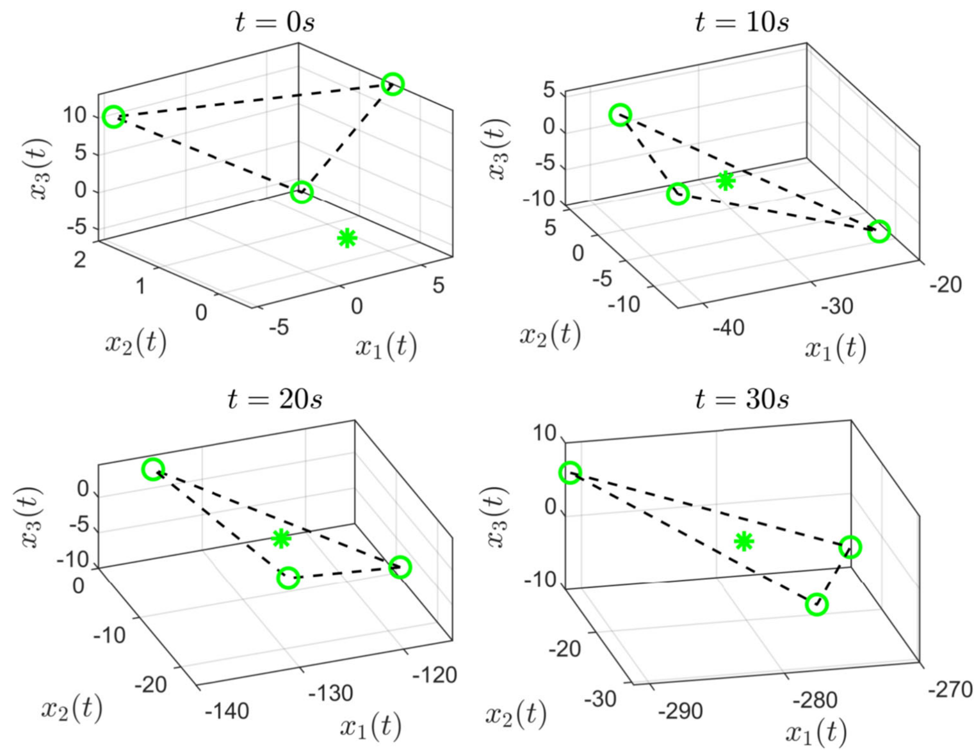

Fig. 5 The position trajectories of agents in subgroup $\mathcal{O}_2$

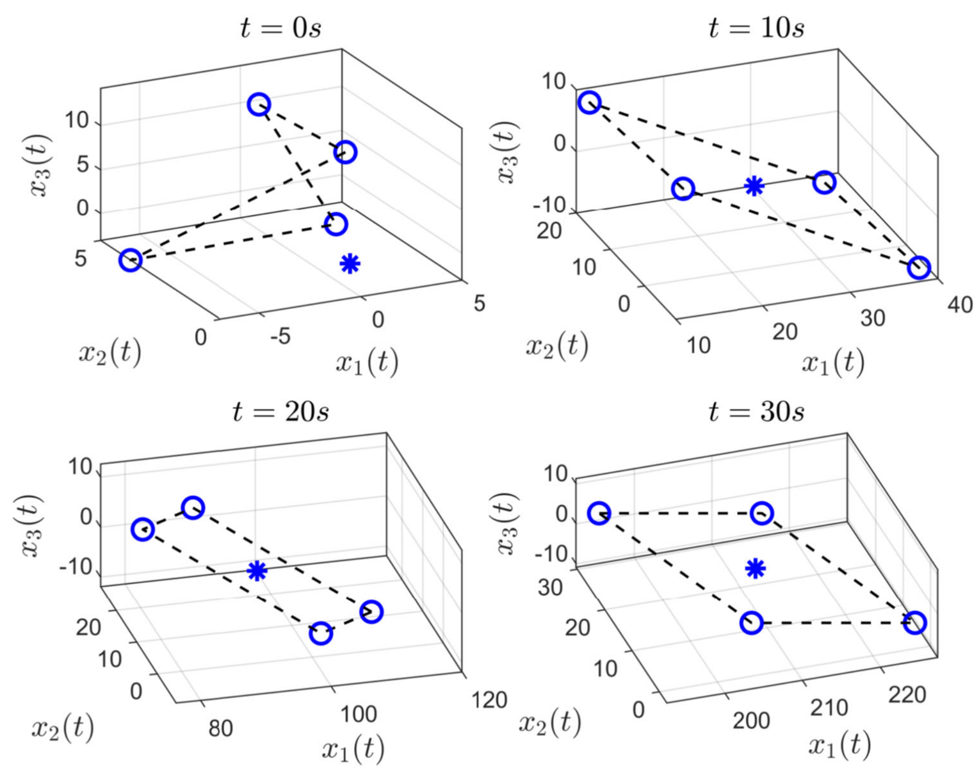

Fig. 6 The position trajectories of agents in subgroup $\mathcal{O}_3$

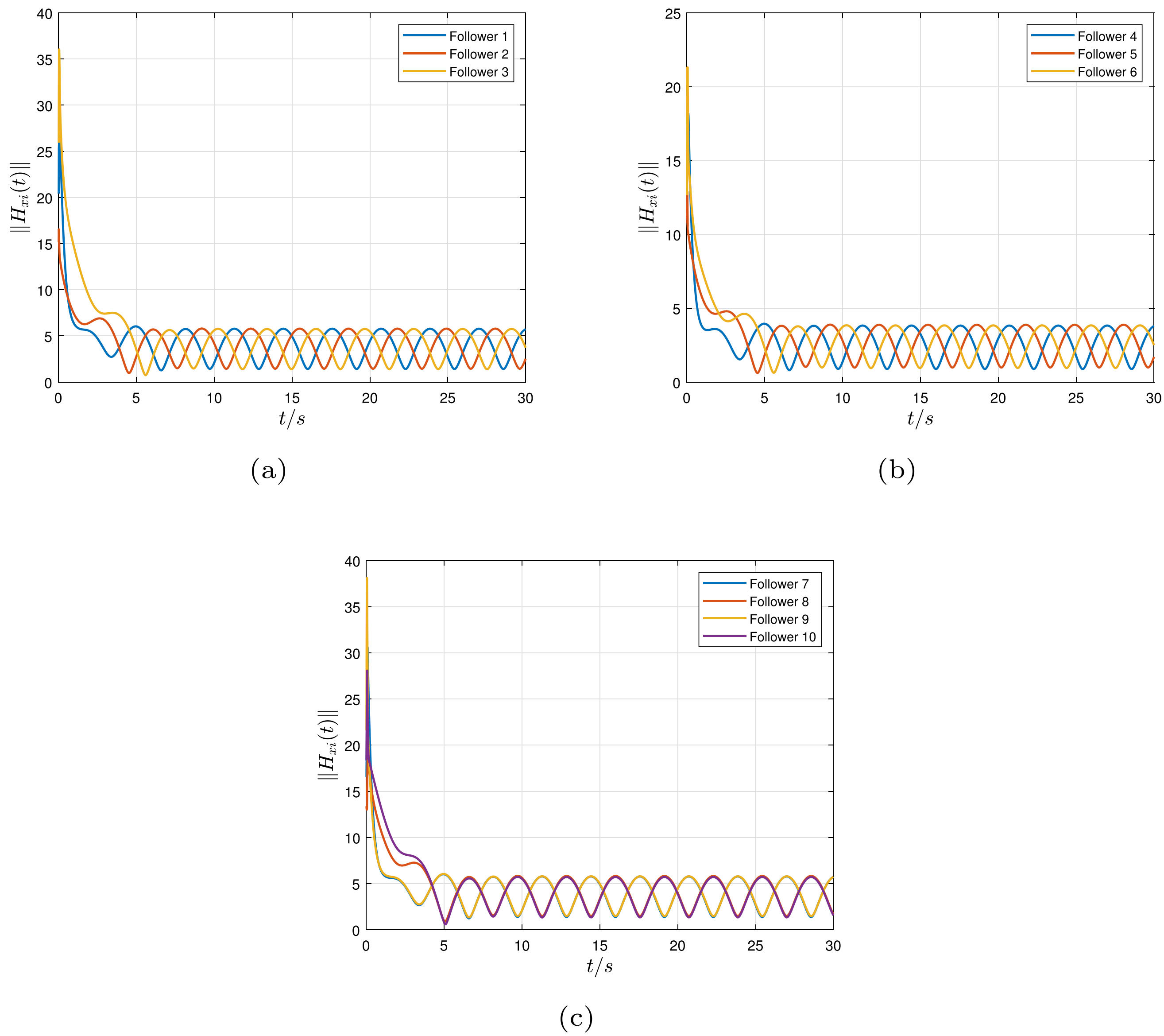

Fig. 7 Distances between the individuals. (a) Subgroup 1. (b) Subgroup 2. (c) Subgroup 3

4.1.3 Numerical comparison

A numerical comparison is further conducted to prove the effectiveness of the proposed collision avoidance feasibility conditions (10). Note that this study focuses on ensuring collision avoidance among followers of the same group. Without loss of generality, let us consider group one as an illustrative example. Within this context, the initial states of followers in group one are configured to satisfy and not satisfy the collision avoidance feasibility conditions (10). Subsequently, the inter-individual distances are depicted in Fig. 8(a) and (b). One sees that if the collision avoidance feasibility conditions (10) are not satisfied, the individuals of group one will collide. However, with appropriate initial state assignments, the individuals within the MASs can maintain sufficient distances, thereby avoiding collisions. In addition, we also compared with the classical artificial potential field collision avoidance method, and the simulation results are shown in Fig. 8(c). Although collision avoidance among agents can be achieved to some extent based on the artificial potential field method, there is still a risk of exceeding the preset safety distance. As pointed out in the existing literature [25, 29], the artificial potential field method has problems such as local minima and difficulty in parameter tuning. On the contrary, the collision avoidance condition designed in this study does not require the addition of a repulsive field and has a more concise form. Thus, the above results further implies the effectiveness of the proposed approach.

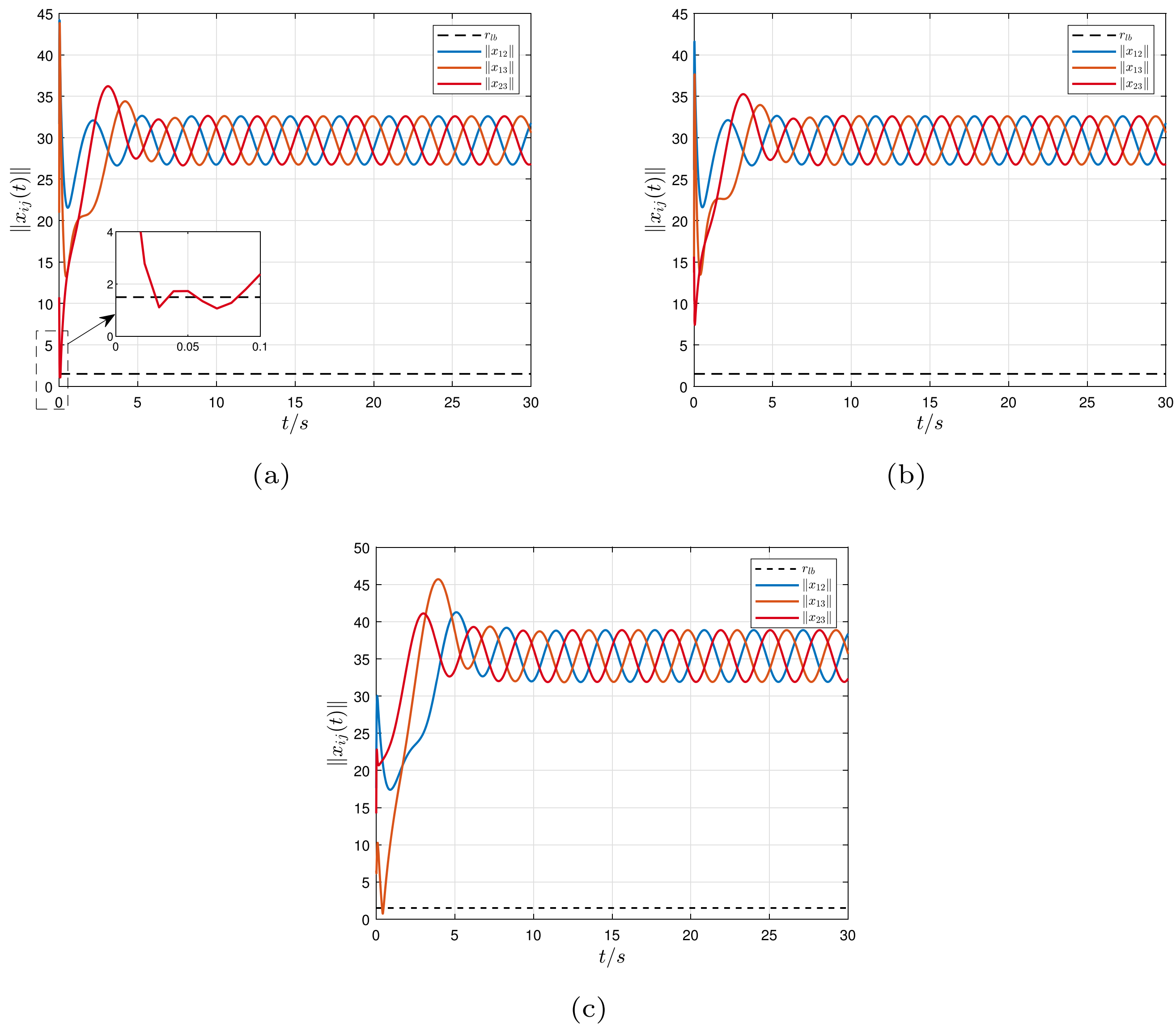

Fig. 8 Distances between the individuals of group one. (a) Not meet the collision avoidance feasibility conditions; (b) Meet the collision avoidance feasibility conditions; (c) Artificial potential field-based collision avoidance

4.2 Time-varying group formation tracking for multi-UAV systems

In this part, the reliability of the formation tracking control scheme designed in this study is verified for the multi-UAV system. Consider a UAV whose dynamics model can be represented by a six-degree-of-freedom (6-DOF) model. Similar to the model considered in [38], this study assumes that all quadrotor UAVs fly at a predetermined constant altitude and that each UAV does not have a formation controller along the Z-axis. Therefore, the formation tracking control problem for MASs can be considered in the two-dimensional XY plane. Based on the outer/inner loop framework established in [39], the dynamics of a quadrotor UAV in the outer loop can be described by the following equation.

$$\begin{cases} \dot{p}_i(t) = v_i(t) \\ \dot{v}_i(t) = \beta_1 p_i(t) + \beta_2 v_i(t) + u_i(t), \end{cases} \tag{21}$$

where $p_i(t) \in \mathbb{R}^n$ and $v_i(t) \in \mathbb{R}^n$ denote the position and velocity vectors of UAV i, respectively, and $u_i(t) \in \mathbb{R}^n$ are the control inputs. Moreover, β_1 and β_2 are two damping constants. Let $x_i(t) = [p_i(t), v_i(t)]^T$, and the UAV system can be rewritten as $\dot{x}_i(t) = Ax_i(t) + Bu_i(t)$, where

$$A = \begin{bmatrix} 0 & 1 \\ \beta_1 & \beta_2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{22}$$

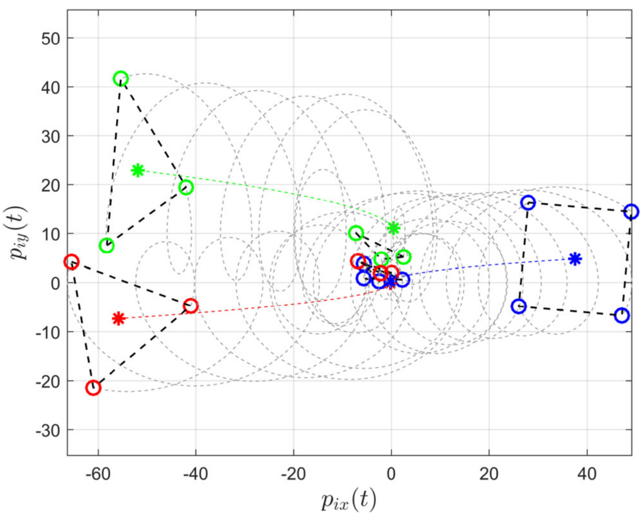

Fig. 9 The state trajectories of each UAV

Consider formation tracking of UAVs in the 2D plane and the relevant time-varying formation parameters are the same as in the previous part. And the damping constants are $\beta_1 = -1$ and $\beta_2 = 0$. The simulation results are shown in Fig. 9. The thirteen UAVs are divided into three groups according to the subgroup relationship, with one leader UAV in each group, and then each group of follower UAV realizes the time-varying formation tracking of the corresponding leader UAV respectively. The reliability and effectiveness of the control scheme designed in this study is further demonstrated.

5 Conclusion

This study has studied the TVGF tracking problem for MASs with collision avoidance. Firstly, the TVGF tracking protocol composed of a feedback controller and formation-related parameters is put forward, where the designing controller can drive the MASs to achieve the TVGF tracking. Then, the conditions to realize the TVGF tracking with collision avoidance are put forward. The convergence of the proposed protocol is analyzed based on the Lyapunov stability theory. Simulations and numerical comparison testify to the effectiveness of the control protocol and collision avoidance feasibility conditions. To sum up, the obtained feasibility conditions ensure collision-free motion without the need for additional forces, distinguishing it from APF-based approaches. This innovation not only simplifies the control strategy but also guarantees the convergence of tracking errors, enhancing efficiency and practicality. Furthermore, by moving away from geometric guidance and constant velocity assumptions, the proposed approach offers a more adaptable solution for diverse real-world scenarios. It is worth noting that in this study, all agents are considered to have the same dynamical model, i.e., the system matrix and the input matrix are the same. In future work, we will further investigate the collision avoidance time-varying group formation tracking control problem for generic linear heterogeneous MASs with multiple leaders.

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Author Contributions Weihao Li and Shiyu Zhou: Writing-original draft. Jiangfeng Yue and Boxian Lin: Software. Mengji Shi and Kaiyu Qin: Supervision.

Data Availability The data-sets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

Declarations

Competing Interests The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this study.

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