Received: 28 September 2020 | Revised: 10 March 2021 | Accepted: 14 April 2021

IET Control Theory & Applications

DOI: 10.1049/cth2.12150

The Institution of Engineering and Technology WILEY

ORIGINAL RESEARCH PAPER

Predefined formation-containment control of high-order multi-agent systems under communication delays and switching topologies

Shiyu Zhou[1] | Xiwang Dong[1,2] | Yongzhao Hua[3] | Jianglong Yu[1] | Zhang Ren[1]

[1] School of Automation Science and Electrical Engineering, Beihang University, Beijing, China

[2] Key Laboratory of Dependable Service Computing in Cyber Physical Society, Chongqing University, Chongqing, China

[3] Institute of Artificial Intelligence, Beihang University, Beijing, China

Correspondence
Xiwang Dong, School of Automation Science and Electrical Engineering, Beihang University, Postal address: No. 37 Xueyuan Road, Haidian District, Beijing 100191, P. R. China.
Email: xwdong@buaa.edu.cn

Funding information
Defense Industrial Technology Development Program, Grant/Award Number: JCKY2019601C106; Foundation Strengthening Program Technology Field Fund, Grant/Award Number: 2019-JCJQ-JJ-243; Science and Technology Innovation 2030 -Key Project of "New Generation Artificial Intelligence", Grant/Award Number: 2020AAA0108200; Key Laboratory of Dependable Service Computing in Cyber Physical Society, Grant/Award Number: CPS-DSC202001; Innovation Zone Project, Grant/Award Number: 18-163-00-TS-001-001-34; National Natural Science Foundation of China, Grant/Award Numbers: 61803014, 61873011, 61922008, 61973013

Abstract

This paper concerns the problem of formation-containment control for general-linear multi-agent systems (MASs) with both communication delays and switching interaction topologies. On the one hand, the leaders can communicate with each other to form the desired formation and on the other, the followers need to enter the convex envelope spanned by the multiple leaders. Firstly, by using the neighbouring relative information, formation-containment protocols are designed for each leader and follower, where an edge-based state observer is incorporated into the formation-containment controller to evaluate the whole leaders' state. Secondly, according to the linear matrix inequality technology, an algorithm is given to determine the unknown feedback matrixes in the protocol. Then, based on Lyapunov theory, the formation-containment error is proved to be convergent and formation feasibility conditions are also presented for the MASs to achieve formation-containment. Finally, a simulation on several MASs is provided to demonstrate the theoretical results.

1 | INTRODUCTION

Recently, due to the benefit of high scalability and low computing consumption, coordinated control of MASs has received considerable attention in various aeries, for example, unmanned aerial vehicles (UAVs) [1, 2], satellite systems [3–6], and under-actuated surface vessels [7]. Moreover, its research fields can be classified into different categories, for instance, consensus control approach [8–13], formation control problem [14–18], containment control approach [19–22], and formation-containment control approach [23–26].

Inspired by the biological swarm systems in real world, formation control, whose goal is to drive the unordered agents to achieve the expected formation, has received considerable attention. Based on consensus approach, Ren et al. in [27] put forward a unique formation control protocol for low-order multi-vehicle systems. However, in practical environment, such as the congestion of the interaction channel and communication range constraints, communication delays and switching topologies emerge. Therefore, time-delayed protocol was presented for the MASs with directed topologies in [28]. Considering a more general systems, by only using the information between the neighbours, a formation problem for high-order MASs with communication delays was investigated in [29]. Note that switching topologies can affect the stability of the systems and cause the chaos of the agents. Therefore, a formation controller of MASs with high-order models under the influence of switching directed networks was studied in [30]. Moreover, a distributed formation approach under both switching topologies and communication delays was proposed and formation feasibility condition was also presented in the work [31]. In [32], a consensus-based approach was presented for the MAS in the presence of communication delays and switching interaction topologies. As a counterpart with formation, another popular research field in coordinated control is containment control, where the agents in this problem are divided into two categories, leaders and followers. The control target is to enforce the followers enter the convex combination spanned by multiple leaders. As a pioneering work in containment, a hybrid Stop-Go policy was presented to drive the followers entering the convex hull in [33]. Considering the time delay, an observer-based containment protocol was studied for high-order MASs with directed graph in [34]. To satisfy the complex environments, both continuous and discrete-time systems were taken into consideration with the influence of communication delays on the containment control in [35]. Note that the interaction topologies of the MASs can be switching due to the uncertain environments. Su et al. in [36] studied the containment problem for MASs with generic linear under switching networks. By using the combination of convex analysis and stochastic process, containment control of MASs with stochastic topologies was studied in [37].

Combined the formation control with containment control, a significant formation-containment problem arises which allows the leaders achieve the desired formation and followers can enter the formation. Note that compared with containment control, there exists also leaders and followers, but the leaders should interact with each other to form the expected formation in the formation-containment problem. As a result of the close relationship between the leaders and the followers, the formation-containment control has been applied in many engineering aeries, such as formation coordinated attack and coordinated penetration. In formation coordinated attack, the unmanned aerial vehicles are denoted as the leaders and the manned fighters are represented as the followers. Leaders are allowed to form the desired formation and the followers can enter the formation. Therefore, when the foes arise, the followers which are much more important can be protected. It should be pointed out that because of the interaction between the leaders and followers, it is difficult to isolate the Laplace matrix between them. Formation-containment control methods and feasible conditions for general-linear swarm systems were investigated in [38], which means the above control methods, no matter it is formation, containment or consensus problem, they all can be thought as a special case of formation-containment problem. The control strategy of formation-containment was applied into the UAVs signifying the effectiveness of the proposed method in the work of Dong et al. [39]. Both fixed and time-varying communication delays were taken into consideration for MASs to achieve formation-containment problems in [40, 41]. For multiple UAVs with directed switching topologies, Han in [42] extends the formation-containment control for MASs to multiple UAVs under directed switching topologies. In the prior works mentioned above, the control strategies for formation-containment are step into a more practical field. However, it cannot be avoided that the communication delays and switching networks may exists simultaneously. Formation-containment problem for MASs with general linear dynamic with communication delays and switching networks is challenging.

In the current paper, a consensus-based solution was put forward to realized formation-containment of generic linear MASs under communication delays and switching topologies. First, because of the different roles that the agents play, two different protocols are proposed for the leaders and follower. Then, necessary and sufficient conditions for the leaders to achieve the expected formation are provided under the constraints of communication delays and switching topologies. Moreover, utilizing the method LMI technology, an approach is developed to obtain the gain matrices. At last, based on the proposed Lyapunov function and formation feasibility condition, both the state observers and formation-containment systems are said to be convergent.

Compared with the outstanding works mentioned above, the main contributions of this paper can be listed as following.

1. Because the leaders have a huge impact on the followers, formation-containment control is more difficult in cooperative control. Compared with formation control in [14–16, 27–29], the followers can enter the formation spanned by the multiple leaders. Besides, in contrast to the containment control in [19, 34–36], the leaders can exchange information with each other to realize the predefined formation. Therefore, the formation-containment problems have important value of engineering application.
2. Note that MASs with communication delays and switching interaction topologies were investigated in the current paper. In the work of Han et al. [41], formation-containment problem with communication delays was studied. Note that if switching topologies emerge, the convex combination of the leaders may change, and the formation-containment error cannot be convergent. Compared with formation-containment problem in [42], the systems are divided into the two systems. It should be pointed out that only switching networks is taken into consideration and the convergence of the systems focuses on each switching system. Therefore, the methods mentioned before can not be directly used to solve the formation-containment systems with both time-varying communication delays and switching networks.
3. The control protocol was studied for general linear MASs on formation-containment. Hence, the formation-containment control mentioned in this paper are practical than the work in [41].

The paper is organized as follows. Section 2 gives some preliminaries on graph theory and the statements of the formation-containment problems. Both control protocols and some important theorems are introduced in Section 3. Then, simulation results which can illustrate the effectiveness of the proposed method are presented in Section 4, Finally, conclusions are demonstrated in Section 5.

The notations throughout this paper are shown as follows. Let $\otimes$ be the Kronecker product. $\mathbf{1}$ stands for a column vector with 1 as its element. I_n represents the $n \times n$ identify matrix. Let $diag\{D_1, D_2, \ldots, D_N\}$ represents a diagonal matrix. X^T denotes the transpose matrix of the matrix X.

2 | PRELIMINARIES AND PROBLEM FORMULATION

In this section, some preliminaries of graph theory are demonstrated and the descriptions on formation-containment are also proposed.

2.1 | Graph theory

A graph $\mathcal{G}$ is denoted to $\{\mathcal{V}, \mathcal{E}, \mathcal{W}\}$, where $\mathcal{V} = \{v_1, v_2, \ldots v_n\}$ is the set of the nodes, $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ represents the edges set, as well as $\mathcal{W} = [w_{ij}] \in \mathbb{R}^{n\times n}$ indicates a weighted adjacency matrix. Let e_{ij}=(v_i, v_j) denotes the edge form node v_i to v_i and w_{ij} is the non-negative element with e_{ji}. If and only if v_j can interaction with v_i, $w_{ij} > 0$; otherwise, $w_{ij} = 0$. Let $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_j, v_i) \in \mathcal{E}\}$ is neighbour set of node v_i. The Laplacian matrix $\mathcal{L} \in \mathbb{R}^{n\times n}$ associated with $\mathcal{G}$ is defined as $\mathcal{L} = \mathcal{D} - \mathcal{W}$, where $\mathcal{D}$ is the in-degree diagonal matrix of the graph. A path form v_i to v_j is defined as an ordered sequence of the edges (v_i, v_{i1}), (v_{i1}, v_{i2}), $\ldots$, (v_{ik}, v_j). The graph is said to be undirected if $v_{ij} \in \mathcal{E}$ implies $v_{ji} \in \mathcal{E}$ and $w_{ij} = w_{ji}$, as well as the connected undirected graph is defined as there exists a path for any two nodes v_i and v_j.

Note that the interaction among the MASs can be switching. The index set of the switching graph is represented by $\mathcal{H} = \{1, 2, \ldots, s\}$. Let $[t_k, t_{k+1})$ denote an infinite sequence of non-overlapping time intervals, where $t_0 = 0$, $t_k - t_{k+1} \geq T_d > 0$. The interaction topology can be switching at the instant t_k and keeps fixed in the dwell time T_d. $\sigma(t) : [0, \infty) \to \{1, 2, \ldots, s\}$ is the switching signal. Define $\mathcal{G}_{\sigma(t)}$ and $\mathcal{L}_{\sigma(t)}$ as the graph and Laplacian matrix at t.

Assume that the neighbours of the leaders are only leader and that there exist M leaders and N followers. Let $\mathcal{O}_E = \{1, 2, \ldots, M\}$ and $\mathcal{O}_F = \{1, 2, \ldots, N\}$ represent, respectively, the leader set and follower set.

The Laplacian matrix $\mathcal{L}$ of the graph at t is given as

$$\mathcal{L}_{\sigma(t)} = \begin{bmatrix} \mathcal{L}^F_{\sigma(t)} & \mathcal{L}^{FE}_{\sigma(t)} \\ 0 & \mathcal{L}^E_{\sigma(t)} \end{bmatrix}$$

where $\mathcal{L}^F_{\sigma(t)} \in \mathbb{R}^{N\times N}$, $\mathcal{L}^{FE}_{\sigma(t)} \in \mathbb{R}^{N\times M}$ and $\mathcal{L}^E_{\sigma(t)} \in \mathbb{R}^{M\times M}$. Denote $\mathcal{G}^F_{\sigma(t)}$ and $\mathcal{G}^E_{\sigma(t)}$ the interaction topology among followers and leaders at t, respectively. Hence, $\mathcal{L}^F_{\sigma(t)}$ and $\mathcal{L}^E_{\sigma(t)}$ are the corresponding Laplacian matrixes.

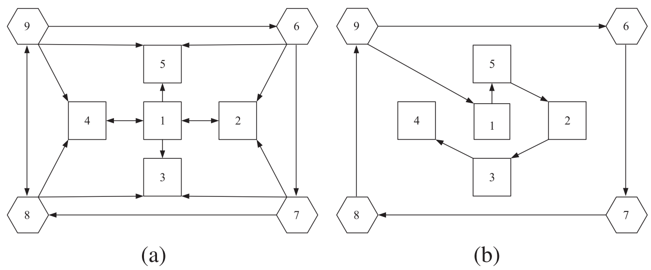

FIGURE 1 Switching topologies in the example

Assumption 1. Suppose that the communication topology $\mathcal{G}^F_{\sigma(t)}$ is connected and undirected.

Assumption 2. For each switching topology, there exist at least one follower who can communicate with all the leaders.

Remark 1. To realize the formation-containment control with both communication delays and switching topologies, the Assumption 1 and Assumption 2 represented before are necessary to construct a state observer (see the illustrative example 1). It should be pointed out that the well-informed follower can be seen in the works [44–46], and in practical applications, the well-informed followers are denoted as the agents with powerful sensors, while the uninformed followers are those with poor sensors. Therefore, the well-informed followers can reach the information of all the leaders and the informed followers can only receive the information of the neighbouring followers.

To show that Assumption 1 and Assumption 2 are important for the MASs to construct a new control framework, by using the control protocol and gain matrices in [40], a numerical example is shown as follows:

Illustrative example 1: Assume there exists a MAS consisting of five followers and four leaders. The time delay satisfies $\tau = 0.15$. The switching topologies are denoted as in Figure 1, and will switch at $t = 10, 30, 50$ s.

The systems matrices are represented as:

$$A = \begin{bmatrix} -3 & 1 & -3 \\ 0 & 0 & 1 \\ -2 & -4 & -6 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Then, the control protocol for the i-th follower is given as:

$$u_i(t) = K_1 x_i(t) + K_2 \sum_{j\in N_i} w_{ij}\big(x_i(t-\tau) - x_j(t-\tau)\big).$$

The control input for i-th leader is given as following:

$$u_i(t) = K_1 x_i(t) + K_3 \sum_{j\in N_i} w_{ij}\Big(\big(x_i(t-\tau) - h_i(t-\tau)\big) - \big(x_i(t-\tau) - h_i(t-\tau)\big)\Big).$$

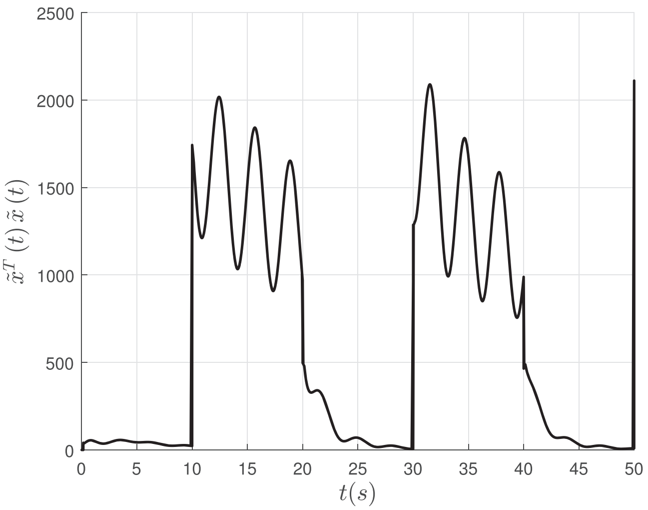

FIGURE 2 Curve of the containment error in the example

Let the gain matrices in the formation-containment control protocol be $K_1 = [-3, 3, 1]$, $K_2 = [0.4978, -0.1074, -0.2669]$, $K_3 = [0.5361, -0.0562, -0.2499]$.

The desired formation for the four leaders is specified by

$$h_i(t) = \begin{bmatrix} 15\sin\left(t + \dfrac{(i-6)\pi}{2}\right) \\ 15\cos\left(t + \dfrac{(i-6)\pi}{2}\right) \\ -15\sin\left(t + \dfrac{(i-6)\pi}{2}\right) \end{bmatrix} \quad (i = 6, 7, 8, 9).$$

From Figure 2, one sees that if the topologies are switching, the convex combinations of the leaders may change, and then the containment error cannot be convergent. Therefore, the exiting formation-containment control protocol in [40–42] no longer meets the circumstance of both time-varying delays and switching topologies. A new control protocol should be put forward to solve the complex communication environment.

Lemma 1. *[8] Assume that assumption 1 holds, 0 is the simple eigenvalue belonging to the $\mathcal{L}^F_{\sigma(t)}$, and its corresponding right eigenvector is $1_N/\sqrt{N}$. The rest of the others $N-1$ eigenvalues are greater that zeros.*

2.2 | Problem description

The agents dynamics are described by

$$\left[\begin{cases} \dot{z}_i(t) = Az_i(t) + Bu_{iE}(t) & i \in \mathcal{O}_E \\ \dot{x}_i(t) = Ax_i(t) + Bu_{iF}(t) & i \in \mathcal{O}_F \end{cases}\right., \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$. $x_i(t) \in \mathbb{R}^{n\times n}$ and $z_i(t) \in \mathbb{R}^{n\times n}$ denote the states of the i-th follower and leaders, respectively. $u_{iE}(t)(i \in \mathcal{O}_F)$ and $u_{iF}(t)(i \in \mathcal{O}_E)$ represent, respectively, the control input of the i-th follower and leaders.

Let $h_E(t) = [h_1^T(t), h_2^T(t) \dots, h_M^T(t)] \in \mathbb{R}^{Mn}$ denote the time-varying formation, where $h_i^T(t)$ is piecewise continuously differentiable.

Definition 1. If for each leader in MASs (1), there exists a vector-value function $f(t) \in \mathbb{R}^n$ satisfying the following equation

$$\lim_{t\to\infty}\left(z_i(t) - f(t) - h_i(t)\right) = 0 \ (i \in \mathcal{O}_E). \tag{2}$$

Then the leaders are proved to achieve the formation $h_E(t)$.

Definition 2. The followers in MASs (1) can realize the containment if (3) holds for each follower

$$\lim_{t\to\infty}\left(x_i(t) - \sum\nolimits_{j=1}^{M} \varepsilon_{ij} z_j(t)\right) = 0 (i \in \mathcal{O}_F), \tag{3}$$

where $\varepsilon_{ij} (i \in \mathcal{O}_F, j \in \mathcal{O}_E)$ are the positive constants with $\sum_{j=1}^{M} \varepsilon_{ij} = 1$.

Definition 3. If both (2) and (3) are satisfying under any given bounded initial states, the MASs (1) can realize the formation-containment.

Remark 2. According to definition 1–3, it can be realized that the formation-containment problem can be seen as the integration of the formation and containment problem. Moreover, it is a much more complex condition on coordinated control. Note that if $N = 0$, which means there is no leaders in MASs. Then, the formation-containment can be transformed into the formation control. If $h_E(t) = 0$ and there is no interaction between the leaders, the formation-containment can be regarded as the containment control. Furthermore, if both N and $f(t)$ are equal to 0, the formation-containment problems discussed in the current paper becomes an state consensus control. Formation-containment control problem arise after the consensus problem and can be regarded as the extension of the consensus control.

Remark 3. Note that although in formation-tracking problem, there exist leaders and followers. It should be pointed out that the agents play different roles comparing with formation-containment control. In formation-containment problem, the leaders should form the expected formation rather than generate the reference trajectory, and the followers should enter the desired formation instead of achieved the predefined formation.

Let $\tilde{z}_i = (z_i(t - \tau(t)) - h_i(t - \tau(t)))(i \in \mathcal{O}_E)$, the formation-containment protocols for leader i $(i \in \mathcal{O}_E)$ is given as following:

$$\begin{aligned} u_{iE}(t) = K_1 z_i(t) + K_2 \sum_{j\in\mathcal{N}^i_{\sigma(t)}} w_{ij}\big[\tilde{z}_j(t - \tau(t)) \\ -\tilde{z}_i(t - \tau(t))\big]. \end{aligned} \tag{4}$$

A state observer and the control input for i-th ($i \in \mathcal{O}_F$) follower are given as

$$\begin{cases} \dot{\hat{\chi}}_i(t) = (I_M \otimes (A + BK_1))\hat{\chi}_i(t) \\ \quad -K_5\left[b_i^{\sigma(t)}\left(\hat{\chi}_i(t-\tau(t)) - z(t-\tau(t))\right) \right. \\ \quad \left. + \sum_{k=M+1}^{M+N} w_{ik}(\hat{\chi}_i(t-\tau(t)) - \hat{\chi}_k(t-\tau(t)))\right], \\ u_{iF}(t) = K_3 x_i(t) - K_4 \sum_{j=1}^{M} \varepsilon_{ij}\hat{\chi}_{ij}(t) \end{cases} \tag{5}$$

where $\hat{\chi}_i(t) = [\hat{\chi}_{i1}^T(t), \hat{\chi}_{i2}^T(t), \ldots, \hat{\chi}_{iM}^T(t)]^T$ with $\hat{\chi}_{i,j}(t)$ representing the evaluate state for j-th leader by follower i. $z(t) = [z_1^T(t), z_2^T(t), \ldots, z_M^T(t)]^T$ denote the compact form of the leaders state, as well as $K_i (i = 1, 2, 3, 4, 5)$ represent the unknown feedback matrices. The predefined containment positive constant is denoted to ε_{ij}, which for each follower i, $\sum_{j=1}^{M} \varepsilon_{ij} = 1$. Suppose that if follower i can communicate with all the leaders in MASs (1), $b_i^{\sigma(t)} > 0$; otherwise, $b_i^{\sigma(t)} = 0$. $\tau(t)$ is the communication delay that satisfy the following assumption. Since all agents have the same control chip and the systems are isomorphic, it makes sense that $\tau(t)$ is the same.

Assumption 3. Let $0 \le \tau(t) \le d_1$ denote the communication delays and the derivate of it is shown as $|\dot{\tau}(t)| \le d_2 < 1$, where $d_2 > 0$.

In order to deal with the influence of the communication delays and switching topologies, the flowing lemmas are proposed.

Lemma 2. *[43] For any continuous derivative vector entry $\ell(t) \in \mathbb{R}^{2d}$, the following is satisfying*

$$\begin{aligned} &-\int_{t-\tau(t)}^{t} \dot{\ell}^T(s) S \dot{\ell}(s) ds \\ &\le \iota^T(t)\begin{bmatrix} M_1^T + M_1 & -M_1^T + M_2 \\ * & -M_2^T - M_1 \end{bmatrix}\iota(t) \\ &+ \tau(t)\iota^T(t)\begin{bmatrix} M_1^T \\ M_2^T \end{bmatrix} S^{-1}[M_1, M_2]\iota(t) \end{aligned}, \tag{6}$$

where $M_1, M_2 \in \mathbb{R}^{2d}$, $S = S^T > 0$ and $\iota(t) = [\ell^T(t), \ell^T(t-\tau(t))]^T$.

Let $H_{\sigma(t)} = B_{\sigma(t)} + \mathcal{L}_{\sigma(t)}^F$, $\sigma(t) \in \{1, 2, \ldots, s\}$, where $B_{\sigma(t)}$ is a diagonal matrix which diagonal element is 0 or 1.

Lemma 3. *[32] According to Assumption 1, the matric $H_{\sigma(t)}$ corresponding to the $\mathcal{G}_{\sigma(t)}^F$ is non-negative.*

Denote by $\lambda_{\sigma(t)}^i$ the eigenvalue of $H_{\sigma(t)}$. Then let $\bar{\lambda}_1 = \min\{\lambda_{\sigma(t)}^i\}$, $\bar{\lambda}_2 = \max\{\lambda_{\sigma(t)}^i\}$, $\sigma(t) \in \{1, 2, \ldots, s\}$.

Lemma 4. *[32] For any $\lambda_{\sigma(t)}^i$ in each switching topology, $\Theta_{\sigma(t)}^i = \Phi_0 + \lambda_{\sigma(t)}^i \Phi_1 < 0$ if and only if $\Theta_i = \Phi_0 + \bar{\lambda}_i \Phi_1 < 0$ ($i \in \{1, 2\}$).*

The major goal of this paper is to solve the following two problems for MASs(1) under both communication delays and switching topologies: First, under what conditions the formation-containment can be realized. Second, how to design the protocols (4) and state observer (5).

3 | MAIN RESULTS

In this section, firstly, a neighbouring-based leaders controller as well as an observer-based followers protocol is put forward for MASs (1) to handle both communication delays and switching graphs. Then, the necessary and sufficient conditions for the leaders to achieve the time-varying formation is proposed. Moreover, an algorithm is given to design the protocols (4) and state observer (5). Finally, the stability of the closed loop MASs (1) is proved by the Lyapunov function and LMI technique.

Let $\tilde{z}(t) = [\tilde{z}_1^T(t), \tilde{z}_2^T(t), \ldots, \tilde{z}_M^T(t)]^T$. Then, the closed loop of the leaders system can be written as following

$$\begin{aligned} \dot{z}(t) &= (I_M \otimes (A + BK_1))z(t) \\ &-\left(\mathcal{L}_{\sigma(t)}^E \otimes BK_2\right)\tilde{z}(t-\tau(t)) \end{aligned}. \tag{7}$$

Under protocol (5), the dynamic of the closed loop of the followers is shown as

$$\dot{x}_i(t) = (A + BK_3)x_i(t) - BK_4 \sum_{j=1}^{M} \varepsilon_{ij}\hat{\chi}_{ij}(t) \ (i \in \mathcal{O}_F). \tag{8}$$

Let $\tilde{\chi}_{ij}(t) = \hat{\chi}_{ij}(t) - z_j(t)$ denote the observing error between i-th observer and j-th leader. Then (8) could be rewritten as follows:

$$\begin{aligned} \dot{x}_i(t) =& (A + BK_3)x_i(t) - BK_4 \sum_{j=1}^{M} \varepsilon_{ij}\tilde{\chi}_{ij}(t) \\ &-BK_4 \sum_{j=1}^{M} \varepsilon_{ij} z_j(t) \end{aligned}. \tag{9}$$

According to (6), one has

$$\begin{aligned} \dot{\tilde{z}}(t) =& (I_M \otimes (A + BK_1))\tilde{z}(t) - \left(\mathcal{L}_{\sigma(t)}^E \otimes BK_2\right)\tilde{z}(t-\tau(t)) \\ &+(I_M \otimes (A + BK_1))h_E(t) - (I_M \otimes I_n)\dot{h}_E(t) \end{aligned}. \tag{10}$$

Denote by $\mathcal{U} = [\bar{u}_1, \bar{u}_2, \ldots, \bar{u}_M]$ an orthogonal constant matrix, where $\bar{u}_1 = 1_M/\sqrt{M}$, then based on Lemma 1, it can be obtained that $\mathcal{U}^T \mathcal{L}_{\sigma(t)}^E \mathcal{U} = diag(0, \tilde{\mathcal{U}}^T \mathcal{L}_{\sigma(t)}^E \tilde{\mathcal{U}})$, where

$\tilde{\mathcal{U}} = [\bar{u}_2, \ldots, \bar{u}_M]$. Let $\theta(t) = (\bar{u}_1^T \otimes I_n)\tilde{z}(t)$ and $\phi(t) = (\tilde{\mathcal{U}}^T \otimes I_n)\tilde{z}(t)$, then the leaders system can be divided into two parts.

$$\begin{aligned}\dot{\theta}(t) = \tilde{A}\theta(t) - \frac{1}{\sqrt{N}}\left(1_N^T \otimes I_n\right)\dot{h}_E(t)\\ + \frac{1}{\sqrt{N}}\left(1_N^T \otimes \tilde{A}\right)h_E(t)\end{aligned}, \tag{11}$$

$$\begin{aligned}\dot{\phi}(t) = (I_{M-1} \otimes \tilde{A})\phi(t) - (\mathcal{U}^T \mathcal{L}_{\sigma(t)}^E \mathcal{U} \otimes BK_2)\phi(t - \tau(t))\\ + (\tilde{U}^T \otimes \tilde{A})h_E(t) - (\tilde{U}^T \otimes I_n)\dot{h}_E(t)\end{aligned}, \tag{12}$$

where $\tilde{A} = A + BK_1$.

Theorem 1. *MASs (1) with communication delays and switching graphs can realize the desired formation $h_E(t)$ if and only if*

$$\lim_{t\to\infty} \phi(t) = 0. \tag{13}$$

Proof. Define $\tilde{z}_c(t) = (\mathcal{U} \otimes I_n)[\theta^T(t), 0]^T$, $\tilde{z}_{\bar{c}}(t) = (\mathcal{U} \otimes I_n)[0, \phi^T(t)]^T$. Note that $[\theta^T(t), 0]^T = e_1 \otimes \theta(t)$, where which first component is 1 and others are 0. Then

$$\tilde{z}_c(t) = (\mathcal{U} \otimes I_n)(e_1 \otimes \theta(t)) = \mathcal{U}e_1 \otimes \theta(t) = 1_M \otimes \frac{\theta(t)}{\sqrt{M}}. \tag{14}$$

It is assumed that $\tilde{z}(t) = (\mathcal{U} \otimes I_n)[\theta^T(t), \phi^T(t)]$. One gets

$$\tilde{z}(t) = \tilde{z}_c(t) + \tilde{z}_{\bar{c}}(t). \tag{15}$$

Note that $\mathcal{U} \otimes I_n$ is non-singular, $\tilde{z}_c(t)$ as well as $\tilde{z}_{\bar{c}}(t)$ are linearly independent vector. Form (14) and (15), it can be obtained that

$$\tilde{z}_{\bar{c}}(t) = \tilde{z}(t) - 1_M \otimes \frac{\theta(t)}{\sqrt{M}}. \tag{16}$$

Since $\tilde{z}(t) = z(t) - h_E(t)$, $\lim_{t\to\infty} \tilde{z}_{\bar{c}}(t) = 0$ is equal to $\lim_{t\to\infty} (z(t) - h_E(t) - 1_M \otimes \frac{\theta(t)}{\sqrt{M}}) = 0$.

Because $\tilde{z}_c(t) = (\mathcal{U} \otimes I_n)[\theta^T(t), 0]^T$ and $\mathcal{U} \otimes I_n$ is non-singular. $\lim_{t\to\infty} \tilde{z}_{\bar{c}}(t) = 0$ is also equal to $\lim_{t\to\infty} \phi(t) = 0$. From Definition 1, the leaders of MASs (1) can realize the formation designed by $h_E(t)$. This completes the proof. □

In the following, an Algorithm with five steps is proposed to determine the protocols and observer.

Algorithm 1. The gain matrixes $K_i (i = 1, 2, 3, 4, 5)$ in the controller and state observer can be designed along the following steps

Step 1: K_1 is chosen to specify the eigenvalue of $A + BK_1$ at the closed left-half complex plane. Note that the controllability of the pair (A, B) can guarantee the existence of the K_1.

Step 2: If there exist positive symmetric matrices R_E, Ω_E, S_E and real matrix $\bar{K}_2$ for any $\bar{\lambda}_i^E (i = 1, 2)$, LMI (17) is feasible, the gain matric $K_2 = \bar{K}_2\Omega^{-1}$; otherwise , the Algorithm stops.

$$\prod(\bar{\lambda}_i^E) = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & 0 & R_E \\ & \Xi_{22} & \Xi_{23} & d_1 S_E & 0 \\ & * & -d_1 S_E & 0 & 0 \\ & * & * & -d_1 S_E & 0 \\ & * & * & * & -\Omega_E \end{bmatrix} < 0, \tag{17}$$

where

$$\begin{aligned}
\Xi_{11} &= R_E\tilde{A}^T + \tilde{A}R_E - \bar{\lambda}_i^E(B\bar{K}_2) - \bar{\lambda}_i^E(B\bar{K}_2)^T - (1 - d_2)\Omega_E,\\
\Xi_{12} &= R_E - \bar{\lambda}_i^E B\bar{K}_2 - (2 - d_2)\Omega_E,\\
\Xi_{13} &= d_1 R_E\tilde{A}^T - d_1\bar{\lambda}_i^E(B\bar{K}_2)^T,\\
\Xi_{22} &= -(3 - d_2)\Omega_E,\\
\Xi_{23} &= -d_1\bar{\lambda}_i^E(B\bar{K}_2)^T,\\
\tilde{A} &= A + BK_1.
\end{aligned}$$

Step 3: Choose suitable K_3 to determine the real part of eigenvalues of $A + BK_3$ are greater than or equal to 0.

Step 4: Choose the suitable K_4, following the equation $K_4 = K_3 - K_1$.

Step 5: If there are positive symmetric matrices R_F, Ω_F, S_F and real matrix $\bar{K}_5$ for any $\bar{\lambda}_i^H (i = 1, 2)$, LMI (18) is met. Then $K_5 = \bar{K}_5\Omega_F^{-1}$; otherwise, the algorithm stops.

$$\prod(\bar{\lambda}_i^F) = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & 0 & R_F \\ * & \Xi_{22} & \Xi_{23} & d_1 S_F & 0 \\ * & * & -d_1 S_F & 0 & 0 \\ * & * & * & -d_1 S_F & 0 \\ * & * & * & * & -\Omega_F \end{bmatrix} < 0, \tag{18}$$

where

$$\begin{aligned}
\Xi_{11} &= R_F\tilde{A}_F^T + \tilde{A}_F R_F - \bar{\lambda}_i^H\bar{K}_5 - \bar{\lambda}_i^H\bar{K}_5^T - (1 - d_2)\Omega_F,\\
\Xi_{12} &= R_F - \bar{\lambda}_i^H\bar{K}_5 - (2 - d_2)\Omega_F,\\
\Xi_{13} &= d_1 R_F\tilde{A}_F^T - d_1\bar{\lambda}_i^H\bar{K}_5^T,\\
\Xi_{22} &= -(3 - d_2)\Omega_F,\\
\Xi_{23} &= -d_1\bar{\lambda}_i^H\bar{K}_5^T,\\
\tilde{A}_F &= I_M \otimes A.
\end{aligned}$$

According to the calculated parameters in Algorithm 1, the following theorem holds.

Theorem 2. *If the formation feasibility condition (19) is fit for the all the leaders and the switching topologies satisfying the Assumptions*

$$(A + BK_1)\left(h_i(t) - h_j(t)\right) - \left(\dot{h}_i(t) - \dot{h}_j(t)\right) \equiv 0. \tag{19}$$

Then, MASs (1) can achieve the formation-containment with both communication delays and switching networks based on the protocols (4) and (5) designed by Algorithm 1.

Proof. The proof of the Theorem 2 can be divided into the following two parts.

Firstly, the leaders can achieve the desired formation.

According to Algorithm 1, the leader can realize the desired formation if and only if $\lim_{t\to\infty}\phi(t)=0$. Then, one obtains that (20) and (21) are the necessary and sufficient conditions conditions for the leaders to achieve formation.

(1) Formation feasibility condition

$$(\tilde{\mathcal{U}}^T\otimes\tilde{A})h_E(t)-(\tilde{\mathcal{U}}^T\otimes I_n)\dot{h}_E(t)\equiv 0 \tag{20}$$

(2) The switching system

$$\dot{\varphi}(t)=(I_{M-1}\otimes\tilde{A})\varphi(t)-(\tilde{\mathcal{U}}^T\mathcal{L}_{\sigma(t)}\tilde{\mathcal{U}}\otimes BK_2)\varphi(t-\tau(t)) \tag{21}$$

is asymptotically stable.

Note that $\mathrm{rank}(B)=m$; it can be obtained that there is a nonsingular matrix $\hat{B}=[B_1{}^T,B_2{}^T]^T$ satisfying $B_1B=I_m$ and $B_2B=0$, where $B_1\in\mathbb{R}^{m\times n}$ and $B_2\in\mathbb{R}^{(n-m)\times n}$.

Subsequently, the condition (20) is shown equivalent to the formation feasibility condition (19).

If (19) holds, one has

$$\left(\mathcal{L}^E_{\sigma(t)}\otimes\tilde{A}\right)h_E(t)-\left(\mathcal{L}^E_{\sigma(t)}\otimes I_n\right)\dot{h}_E(t)\equiv 0. \tag{22}$$

Recall that $\mathcal{L}^E_{\sigma(t)}=\mathcal{U}\,diag(0,\tilde{\mathcal{U}}^T\mathcal{L}^E_{\sigma(t)}\tilde{\mathcal{U}})\mathcal{U}^T$; then multiplying both sides of (22) by $\mathcal{U}^T\otimes I_n$, it has

$$\begin{aligned}\left(\left(\tilde{\mathcal{U}}^T\mathcal{L}^E_{\sigma(t)}\tilde{\mathcal{U}}\right)\otimes I_n\right)&\left[\left(\mathcal{U}^T\otimes\tilde{A}\right)h_E(t)\right.\\ &\left.-\left(\mathcal{U}^T\otimes I_n\right)\dot{h}_E(t)\right]\equiv 0\end{aligned}. \tag{23}$$

Moreover, multiplying both sides of (23) by $(\tilde{\mathcal{U}}^T\mathcal{L}^E_{\sigma(t)}\tilde{\mathcal{U}})^{-1}\otimes I_n$, one gets condition (20). It can be verified that (19) is the sufficient condition for (20).

Now lets turn to the necessity. Recall that $\mathrm{rank}(\tilde{\mathcal{U}}^T)=M-1$. Therefore let $\tilde{\mathcal{U}}^T=[\hat{u},\hat{\mathcal{U}}]$, where $\hat{u}\in\mathbb{R}^{(M-1)\times 1}$ and $\hat{\mathcal{U}}\in\mathbb{R}^{(M-1)\times(M-1)}$. If (20) holds, it has

$$\left(\left[\hat{u},\hat{\mathcal{U}}\right]\otimes\tilde{A}\right)h_E(t)-\left(\left[\hat{u},\hat{\mathcal{U}}\right]\otimes I_n\right)\dot{h}_E(t)\equiv 0. \tag{24}$$

Due to $\tilde{\mathcal{U}}1_M=0$, it has $\hat{u}=-\hat{\mathcal{U}}1_{M-1}$. Let $\hat{h}_E(t)=[h_2^T(t), h_3^T(t),\dots,h_M^T(t)]^T$, then it follows from (24) that

$$\left(\hat{\mathcal{U}}\otimes I_n\right)\left(\bar{\Upsilon}-\tilde{\Upsilon}\right)\equiv 0, \tag{25}$$

where

$\bar{\Upsilon}=(I_{M-1}\otimes\tilde{A})\hat{h}_E(t)-(I_{M-1}\otimes I_n)\dot{\hat{h}}_E(t)$,

$\tilde{\Upsilon}=(I_{M-1}\otimes\tilde{A})h_1(t)-(I_{M-1}\otimes I_n)\dot{h}_1(t)$.

Multiplying both sides of (25) by $\hat{\mathcal{U}}^{-1}\otimes I_n$, one has

$$\tilde{A}h_{i1}(t)-\dot{h}_{i1}(t)\equiv 0, \tag{26}$$

where $i=2,3,\dots,M$.

From (26), it can be proved that for $\forall i\in\mathcal{O}_E$ and $j\in\mathcal{N}^i_{\sigma(t)}$, $\tilde{A}h_{ij}(t)-\dot{h}_{ij}(t)\equiv 0$, which means condition (19) holds. Therefore, condition (19) is equivalent to condition (20).

Consider the stability of the system (21). Construct the following common Lyapunov–Krasovskii candidate function

$$V_E(t)=V_{E1}(t)+V_{E2}(t)+V_{E3}(t), \tag{27}$$

where

$$\begin{aligned}V_{E1}(t)&=\varphi^T(t)\left(I_{M-1}\otimes R_E^{-1}\right)\varphi(t),\\ V_{E2}(t)&=\int_{t-\tau(t)}^t\varphi^T(s)\left(I_{M-1}\otimes\Omega_E^{-1}\right)\varphi(s)ds,\\ V_{E3}(t)&=\int_{-d_1}^0\int_{t+\mu}^t\dot{\varphi}^T(s)\left(I_{M-1}\otimes S_E^{-1}\right)\dot{\varphi}(s)dsd\mu.\end{aligned}$$

Let $\Lambda_{\sigma(t)}=diag(\lambda^1_{\sigma(t)},\lambda^2_{\sigma(t)},\dots,\lambda^N_{\sigma(t)})$ as it was mentioned that $\tilde{\mathcal{U}}^T\mathcal{L}^E_{\sigma(t)}\tilde{\mathcal{U}}$ is symmetric; therefore, it is possible to get an orthogonal matrix $\tilde{\mathcal{U}}_{\sigma(t)}$ which meeting $\tilde{\mathcal{U}}^T_{\sigma(t)}\tilde{\mathcal{U}}^T\mathcal{L}^E_{\sigma(t)}\tilde{\mathcal{U}}\tilde{\mathcal{U}}_{\sigma(t)}=\Lambda^E_{\sigma(t)}$.

Let $\ell(t)=(\tilde{\mathcal{U}}^T_{\sigma(t)}\otimes I_n)\varphi(t)=[\ell_2^T(t),\ell_3^T(t),\dots,\ell_M^T(t)]^T$, and $\hat{\ell}_i(t)=[\ell_i^T(t),\ell_i^T(t-\tau(t))]^T$; then taking the time derivative of $V_E(t)$ along the trajectory of (27), one has

$$\dot{V}_{E1}(t)=\sum_{i=2}^M\hat{\ell}_i^T(t)\begin{bmatrix}R_E{}^{-1}\tilde{A}+\tilde{A}^TR_E{}^{-1} & -\lambda^i_{\sigma(t)}R_E{}^{-1}BK_3\\ * & 0\end{bmatrix}\hat{\ell}_i(t), \tag{28}$$

$$\begin{aligned}\dot{V}_{E2}(t)&=\ell^T(t)\left(I_{M-1}\otimes\Omega_E^{-1}\right)\ell(t)\\ &-\left(1-\dot{\tau}(t)\right)\ell^T(t-\tau(t))\left(I_{M-1}\otimes\Omega_E^{-1}\right)\ell(t-\tau(t))\end{aligned}. \tag{29}$$

From the Assumption 3, one gets

$$\begin{aligned}\dot{V}_{E2}(t)&\le\ell^T(t)\left(I_{M-1}\otimes\Omega_E^{-1}\right)\ell(t)\\ &-(1-d_2)\ell^T(t-\tau(t))\left(I_{M-1}\otimes\Omega_E^{-1}\right)\ell(t-\tau(t))\\ &=\sum_{i=2}^M\hat{\ell}_i^T(t)\begin{bmatrix}\Omega_E^{-1} & 0\\ 0 & -(1-d_2)\Omega_E^{-1}\end{bmatrix}\hat{\ell}_i(t)\end{aligned}, \tag{30}$$

$$\begin{aligned}\dot{V}_{E3}(t)&=d_1\dot{\ell}^T(t)\left(I_{M-1}\otimes S_E^{-1}\right)\dot{\ell}(t)\\ &-\int_{t-d_1}^t\dot{\ell}^T(s)\left(I_{M-1}\otimes S_E^{-1}\right)\dot{\ell}(s)ds\end{aligned}. \tag{31}$$

Let $\varpi_i=[\tilde{A},-\lambda^i_{\sigma(t)}BK_3]$, then

$$\begin{aligned}\dot{\ell}^T(t)\left(I_{M-1}\otimes S_E^{-1}\right)\dot{\ell}(t)&=\ell^T(t)\left(I_{M-1}\otimes\tilde{A}S_E^{-1}\tilde{A}\right)\ell(t)\\ &-2\ell^T(t)\left(\Lambda_{\sigma(t)}\otimes\tilde{A}^TS_E^{-1}BK_3\right)\ell(t-\tau(t))\\ &+\ell^T(t-\tau(t))(\Lambda^2_{\sigma(t)}\otimes(BK_3)^TS_E^{-1}(BK_3))\ell(t)\\ &=\sum_{i=2}^M\hat{\ell}_i^T(t)\varpi_i^TS_E^{-1}\varpi_i\hat{\ell}_i(t).\end{aligned} \tag{32}$$

From Assumption 3 and Lemma 2, it can be obtained that

$$\begin{aligned}
&-\int_{t-d_1}^{t} \dot{e}^T(s)(I_{M-1}\otimes S_E^{-1})\dot{e}(s)ds \\
&\quad\le -\int_{t-\tau(t)}^{t} \dot{e}^T(s)(I_{M-1}\otimes S_E^{-1})\dot{e}(s)ds \\
&\quad= -\int_{t-\tau(t)}^{t}\sum_{i=2}^{M} e_i^T(t)S_E^{-1}e_i(t)ds \\
&\quad= \sum_{i=2}^{M}\left(-\int_{t-\tau(t)}^{t} e_i^T(t)S_E^{-1}e_i(t)ds\right) \\
&\quad\le \sum_{i=2}^{M}\hat{e}_i^T(t)\left(\begin{bmatrix} M_1^T+M_1 & -M_1^T+M_2 \\ * & -M_2^T-M_2\end{bmatrix}\right. \\
&\qquad\left.+d_2\begin{bmatrix} M_1^T \\ M_2^T\end{bmatrix}S_E^{-1}[M_1,M_2]\right)\hat{e}_i(t)
\end{aligned} \quad . \tag{33}$$

Let $M_1=-R_E^{-1}$, $M_2=\Omega_E^{-1}$, from (27) to (33), it can be obtained that

$$\dot{V}_E(t)\le\sum_{i=2}^{M}\hat{e}_i^T(t)Z_i\hat{e}_i(t), \tag{34}$$

where

$$Z_i=T_i+d_1\varpi_i^T S_E^{-1}\varpi_i+d_1\begin{bmatrix}-R_E^{-T}\\ \Omega_E^{-T}\end{bmatrix}S_E^{-1}\left[-R_E^{-1},\Omega_E^{-1}\right],$$

$$T_i=\begin{bmatrix} T_{i11} & \Omega_E^{-1}+R_E^{-1}-\lambda_{\sigma(t)}^{i}R_E^{-1}BK_3 \\ * & -(3-d_2)\Omega_E^{-1}\end{bmatrix},$$

$$T_{i11}=-2R_E^{-1}+R_E^{-1}\tilde{A}+\tilde{A}^TR_E^{-1}+\Omega_E^{-1}.$$

According to Schur complement lemma, $Z_i<0$ is equivalent to $\Psi_i<0$

$$\psi_i=\begin{bmatrix} T_i & d_1\varpi_i^T & d_1\left[-R_E^{-1} \quad -\Omega_E^{-1}\right] \\ & d_1S_E^{-1} & 0 \\ & * & -d_1S_E^{-1}\end{bmatrix}<0$$

Let $\Gamma=[\begin{smallmatrix} R_E & 0 \\ \Omega_E & \Omega_E\end{smallmatrix}]$ and $\bar{\Gamma}=diag\{T,I,S\}$, then it can be obtained that

$$\bar{\Gamma}^T\psi_i\bar{\Gamma}=\begin{bmatrix}\Gamma^T T\Gamma_i & d_1\Gamma^T\varpi_i^T & d_1\left[0 \quad S_E\right] \\ & d_1S_E & 0 \\ & * & -d_1S_E\end{bmatrix}$$

As $K_2=\bar{K}_2\Omega^{-1}$, $\prod(\bar{\lambda}_i^E)<0$; then according to Schur complement lemma, $\prod(\bar{\lambda}_i^E)<0$ are equivalent to $\prod(\lambda_{\sigma(t)}^i)<0(i=2,3,\dots,M,\sigma(t)=1,2,\dots,s)$, which equal to $\bar{\Gamma}^T\psi_i\bar{\Gamma}<0$, from (24) to (34), one has

$$\lim_{t\to\infty}\phi(t)=0. \tag{35}$$

Remember (19) is the necessary and sufficient condition of (20). From Schur complement lemma and the calculated K_2, one gets $\prod(\bar{\lambda}_i^E)<0$; then based on Lemma 4, $\prod(\bar{\lambda}_i^E)<0$ are equivalent to $\prod(\lambda_{\sigma(t)}^i)<0(i=2,3,\dots,M,\sigma(t)=1,2,\dots,s)$ for each switching systems, which means the convergence of the formation error systems (35) is proved and MASs (1) under both switching graphs and communication delays realized the desired formation $h_E(t)$.

The follower will enter the predefined convex combination by the leaders.

Note that $K_4=K_3-K_1$. Then let $\tilde{\chi}_i(t)=\hat{\chi}_i(t)-z(t)$. According to (5), one has

$$\begin{aligned}
\dot{\tilde{\chi}}_i(t)&=(I_M\otimes(A+BK_3))\tilde{\chi}_i(t)-K_5\Big[b_i^{\sigma(t)}\tilde{\chi}_i(t-\tau(t)) \\
&\quad+\sum_{k=M+1}^{M+N}\omega_{ik}\big(\tilde{\chi}_i(t-\tau(t))-\tilde{\chi}_k(t-\tau(t))\big)\Big] \\
&\quad+\mathcal{L}_{\sigma(t)}^E\otimes BK_2 z(t-\tau(t))
\end{aligned} \quad . \tag{36}$$

Let $\iota(t)=[\tilde{\chi}_1^T(t),\tilde{\chi}_2^T(t),\dots,\tilde{\chi}_N^T(t)]^T$ and $\tilde{A}=I_M\otimes(A+BK_3)$, then (36) could be rewritten in a compact form as follows

$$\begin{aligned}
\dot{\iota}(t)&=(I_N\otimes\tilde{A})\iota(t)-(H_{\sigma(t)}\otimes K_5)\iota(t-\tau(t)) \\
&\quad+I_N\otimes\left(\mathcal{L}_{\sigma(t)}^E\otimes BK_2 z(t-\tau(t))\right)
\end{aligned} , \tag{37}$$

where $H_{\sigma(t)}=B_{\sigma(t)}+\mathcal{L}_{\sigma(t)}^F$. $\mathcal{L}_{\sigma(t)}^F$ is followers symmetric Laplacian matrix at t, and $B_{\sigma(t)}=diag[b_1^{\sigma(t)},b_2^{\sigma(t)},\dots,b_N^{\sigma(t)}]$.

Consider the stability of system (37). As shown in [39], since $\tilde{z}_i(t)=z_i(t)-h_i(t)(i\in\mathcal{O}_E)$, $\tilde{z}(t)=[\tilde{z}_1^T(t),\tilde{z}_2^T(t),\dots,\tilde{z}_M^T(t)]^T$; therefore, $\lim_{t\to\infty}\mathcal{L}_{\sigma(t)}^E\otimes BK_2\tilde{z}(t-\tau(t))=0$, the stability of system (37) is equivalent to the following system (38)

$$\dot{\iota}(t)=(I_N\otimes\tilde{A})\iota(t)-(H_{\sigma(t)}\otimes K_5)\iota(t-\tau(t)). \tag{38}$$

The stability analysis of (38) is similar to (21), which means as $t\to\infty$, the state observed errors $\tilde{\chi}_i(t)(i=1,2,\dots,N)$ can converge to zeros.

From (1) and (4), one has

$$\begin{aligned}
\dot{z}_i(t)&=(A+BK_1)z_i(t)+ \\
&BK_2\sum_{j\in\mathcal{N}_{\sigma(t)}^i}w_{ij}\left[\tilde{z}_j(t-\tau(t))-\tilde{z}_i(t-\tau(t))\right].
\end{aligned} \tag{39}$$

Let containment error $\tilde{x}_i(t)=x_i(t)-\sum_{j=1}^{M}\varepsilon_{ij}z_j(t)$. It follows from (9) and (39) that

$$\begin{aligned}
\dot{\tilde{x}}_i(t)&=(A+BK_3)x_i(t)-BK_4\sum_{j=1}^{M}\varepsilon_{ij}\tilde{\chi}_{i,j}(t) \\
&-(A+BK_1+BK_4)\sum_{j=1}^{M}\varepsilon_{ij}z_j(t) \\
&-\sum_{j=1}^{M}\varepsilon_{ij}BK_2\sum_{k\in\mathcal{N}_{\sigma(t)}^j}w_{jk}\left[\tilde{z}_k(t-\tau(t))-\tilde{z}_j(t-\tau(t))\right]
\end{aligned} \tag{40}$$

As $t \to \infty$, based on Algorithm 1, leaders can form the formation. Therefore, $\lim_{t\to\infty}[\tilde{z}_k(t-\tau(t)) - \tilde{z}_j(t-\tau(t))] = 0$. Moreover, according to Algorithm 1, $K_4 = K_3 - K_1$. Then (40) can be transformed into

$$\dot{\tilde{x}}_i(t) = (A + BK_3)\tilde{x}_i(t) - BK_4 \sum_{j=1}^{M} \epsilon_{ij}\tilde{\chi}_{ij}(t). \tag{41}$$

From Algorithm 1 and (38), the gain matrix K_3 is chosen to make $A + BK_3$ Hurwitz. $\tilde{\chi}_i(t) = [\tilde{\chi}_{i1}^T(t), \tilde{\chi}_{i2}^T(t), \dots, \tilde{\chi}_{iM}^T(t)]^T$, which means $\lim_{t\to\infty} \tilde{\chi}_{ij}^T(t) = 0 (j = 1, 2, \dots, M)$. Therefore, according to input-to-state stability in [52], it can be verified that $\lim_{t\to\infty} \dot{\tilde{x}}_i(t) = 0$, which implies containment error can converge to zero.

From (35) and (41) , MASs (1) can achieve formation-containment control under switching topologies and time-varying delays. This completes the proof. □

Remark 4. Note that the systems (21) and (38) can be viewed as the time-delay switched system in control theory, where we can see that both time delays and switching graphs will have significant influences on the stability of this system. On the one hand, the proposed Lyapunov–Krasovskii function for just delayed systems on fixed topology can no longer be used in the switching case; on the other hand, time-delayed factor is incorporated into the Lyapunov function compared with the switched systems without time delay. Then, by putting forward the useful lemmas and assumptions, we have constructed the common Lyapunov–Krasovskii candidate function (27) to deal with both time delays and switching graphs. Finally, the time-delay switched system is proven to be asymptotic stability.

Remark 5. Based on the stated observer, a formation-containment protocol is proposed for MASs (1), where the pre-defined weights ϵ_{ij} are used to specify the desired combination. One sees that MASs (1) can realize formation-containment with both communication delays and switching topologies. Moreover, only the formations satisfying (19) can be achieved by leaders.

Remark 6. Although there exist some works that the distributed estimation can be achieved in [11], it should be pointed out that the proposed state observer is constructed without neither time-varying delays nor switching topologies, which means the proposed Lyapunov function cannot be used to prove the convergency of the time-delay switched systems, and the analysis method based on orthogonal transformation is only satisfied for fixed topology rather than switching topologies. Moreover, the state observer is put forward to estimate all the leaders' state rather than a single leader's state under both time-varying delays and switching topologies. Therefore, if $b_i^{\sigma(t)} > 0$, the follower can get all the information of the leaders. Due to the physical meaning of the state observer and the coefficient $b_i^{\sigma(t)}$, the follower i is denoted by the well-informed follower.

Remark 7. Note that the distributed observers are also asymptotically stable in the existing results [48–51]. Although the estimation errors cannot converge to zero in finite time, we can design appropriate gain matrices in the observer such that the estimation process is faster than the control process, and then the separation principle is applicable to the closed-loop system. Specifically, in [47], the containment problems with switching topologies are solved based on the state observer systems; an edge-based adaptive distributed observer is presented for each follower to estimate the whole states of all the leaders. The common Lyapunov theory is put forward to prove the convergence of the distributed observers under the influences of switching graphs. Moreover, in the research of the cooperative output regulation [48–51], by using the neighbours' information, the state observer is contrasted to estimate the leaders' state. Then, the control protocol is designed based on the state observer, and the state observers in [48–51] are proven to have asymptotic convergence. Therefore, the asymptotic convergence of distributed observer is appropriate and can guarantee the stability of the whole closed-loop system.

Remark 8. The works in [13] and [35] investigated, respectively, the formation or containment problem under both communication delays and switching topologies. Note that because of the influence of the formation on the followers, the mentioned protocol cannot be used directly to solve the MASs with both communication delays and switching topologies. By using the neighbours information and state observers, a novel protocol is put forward for the MASs (1) to realize formation-containment under both communication delays and switching topologies. Hence, the proposed controller is more versatile and general.

4 | NUMERICAL SIMULATIONS

In this section, numerical simulations are presented to validate the effectiveness of the proposed approach.

Assume that there exist third-order MASs with nine agents consisting of six leaders and three leaders.

$$A = \begin{bmatrix} -3 & 1 & -3 \\ 0 & 0 & 1 \\ -2 & -4 & -6 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Suppose that $\tau(t) = 0.05 + 0.01\cos(t)$, Figure 1 shows the interaction topologies with 0-1 weight. Suppose that interaction topologies are randomly chosen form fig 1 with interval.

The formation for the leaders are defined as

$$h_i(t) = \begin{bmatrix} 3\sin\left(t + \frac{(i-1)\pi}{3}\right) \\ 3\cos\left(t + \frac{(i-1)\pi}{3}\right) \\ -3\sin\left(t + \frac{(i-1)\pi}{3}\right) \end{bmatrix} (i = 1, 2, 3, 4, 5, 6)$$

From $h_i(t)$ $(i = 1, 2, 3, 4, 5, 6)$, it can be obtained that if the formation is realized, the state of the four leader can achieve

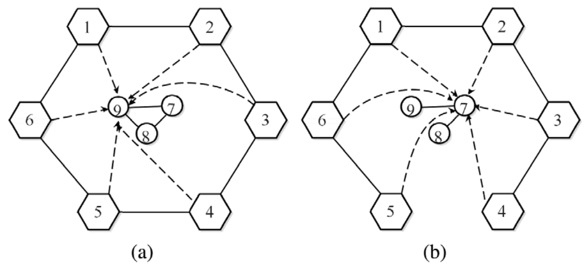

FIGURE 3 Switching topologies

a regular hexagon formation with a radius $r = 3$ m and rotate with a fixed angular velocity $\omega = 1$ rad/s.

According to Algorithm 1, K_1 can be chosen as $[-3\ 3\ 1]$ and the eigenvalues of $A + BK_1$ are specified at -8, j and j. Choose $K_3 = [2\ 2\ 3]$ to place the eigenvalues of $A + BK_3$ at $-1, -2$ and -3. Since $K_4 = K_3 - K_1$, $K_4 = [5\ -1\ 2]$, using Step 2 and Step 5 in Algorithm 1, K_2 and K_5 can be given as follows:

$$K_2 = \begin{bmatrix} -0.6727 & 0.2527 & 0.3983 \end{bmatrix}$$

$$K_5 = I_6 \otimes \begin{bmatrix} 0.1318 & 0.0723 & -0.3503 \\ 0.0743 & 0.4532 & 0.0641 \\ -0.3874 & 0.0225 & -0.1770 \end{bmatrix}$$

The initial state vectors of the agents and the state observers are described by $z_{ij}(0) = 2(\Theta - 0.5)(i = 1, 2, 3, 4, 5, 6; j = 1, 2, 3)$; $x_{ij}(0) = 4(\Theta - 0.5)(i = 7, 8, 9; j = 1, 2, 3)$; $\hat{\chi}_{ij}(0) = 2(\Theta - 0.5)(i = 7, 8, 9; j = 1, \ldots, 18)$, where Θ is a random value with a uniform distribution on the interval $(0, 1)$.

The desired state containment for third-order MASs is specified by $\varepsilon_{71} = \frac{1}{12}, \varepsilon_{72} = \frac{1}{12}, \varepsilon_{73} = \frac{1}{6}, \varepsilon_{74} = \frac{1}{6}, \varepsilon_{75} = \frac{1}{4}, \varepsilon_{76} = \frac{1}{4}, \varepsilon_{81} = \frac{1}{6}, \varepsilon_{82} = \frac{1}{6}, \varepsilon_{83} = \frac{1}{3}, \varepsilon_{84} = \frac{1}{8}, \varepsilon_{85} = \frac{1}{8}, \varepsilon_{86} = \frac{1}{12}, \varepsilon_{91} = \frac{1}{4}, \varepsilon_{92} = \frac{1}{4}, \varepsilon_{93} = \frac{1}{12}, \varepsilon_{94} = \frac{1}{12}, \varepsilon_{95} = \frac{1}{6}, \varepsilon_{96} = \frac{1}{6}$.

Figure 4 shows the state snapshots of nine agents, where the states of the leaders and followers are denoted by red and blue, respectively. The imaginary line is shown to demonstrate the convex combination, and the black pentagram denotes the state of the formation reference. Figure 4 indicates that the leaders achieve the desired formation and the followers converge to the convex hull formed by the leaders.

Figure 5 displays the time-varying formation error within $t = 50$ s. As shown in Figures 6 and 7, all the followers can acquire the leaders state and the containment errors can converge to zero. From Figures 2–5, one sees that MASs with time-varying delays and switching interaction topologies achieves the desired formation-containment.

5 | CONCLUSION

Formation-containment problems for general linear MASs with both communication delays and switching interaction topolo-

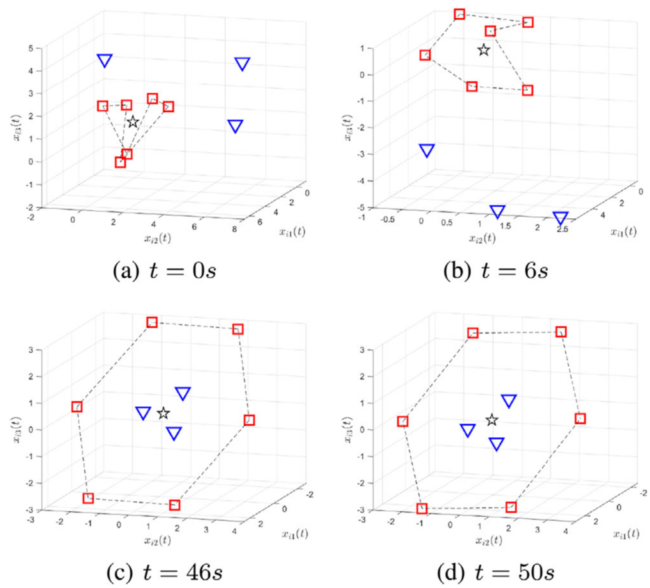

FIGURE 4 Snapshots of nine agents

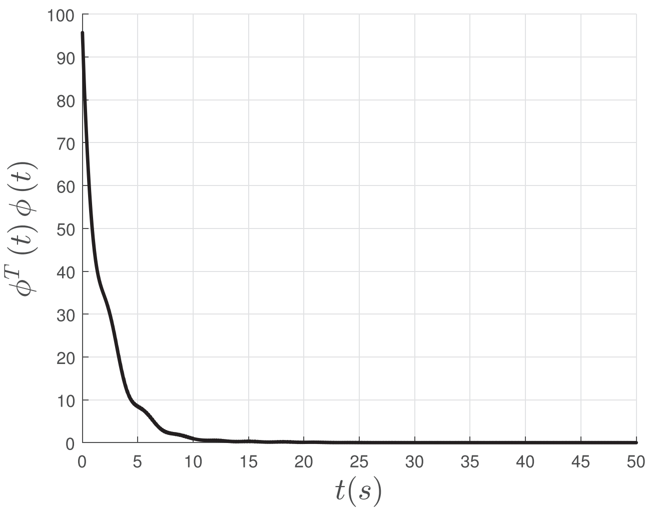

FIGURE 5 Curve of the formation error

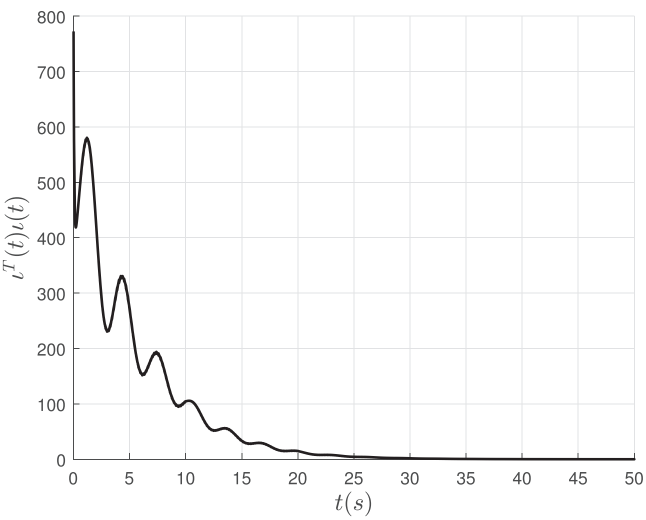

FIGURE 6 Curve of the state observers' error

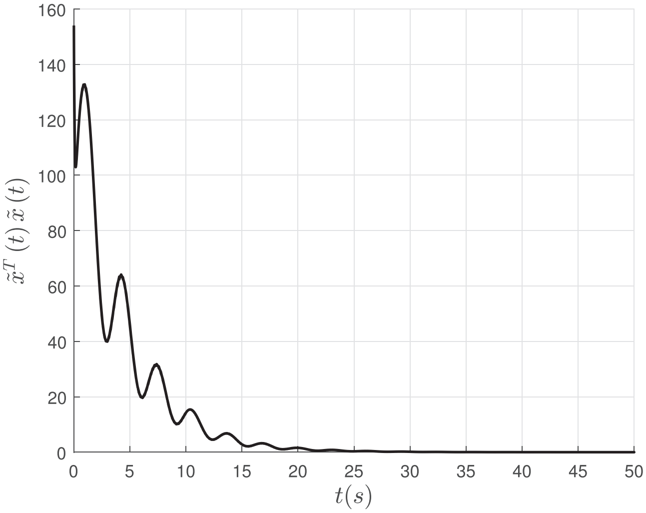

FIGURE 7 Curve of the containment error

gies were studied. The target of the leaders was to achieve the desired formation and the followers should enter the convex combination spanned by the leaders. Based on distributed state observer and neighbouring information, novel control protocols, which considered switching graphs and communication delays, were proposed for the leaders and followers, respectively. According to LMI technique, an approach to determine the feedback matrixes was put forward. Moreover, based on Lyapunov–Krasovskii stability theory and the proposed formation feasibility conditions, the stability of the formation-containment systems was proved. Future research will concentrate on extending the switching topologies from undirected to directed without well-informed follower. Another interesting topic for future study is to design the finite time state observer on the formation-containment control.

ACKNOWLEDGEMENTS

This work was supported by the Science and Technology Innovation 2030-Key Project of "New Generation Artificial Intelligence" under Grant 2020AAA0108200, the National Natural Science Foundation of China under Grants 61922008, 61873011, 61973013 and 61803014, the Defense Industrial Technology Development Program under Grant JCKY2019601C106, the Innovation Zone Project under Grant 18-163-00-TS-001-001-34, the Foundation Strengthening Program Technology Field Fund under Grant 2019-JCJQ-JJ-243, and the Fund from Key Laboratory of Dependable Service Computing in Cyber Physical Society under Grant CPS-DSC202001.

ORCID

Shiyu Zhou https://orcid.org/0000-0003-3742-4264
Xiwang Dong https://orcid.org/0000-0002-4778-248X
Yongzhao Hua https://orcid.org/0000-0003-1449-0795
Zhang Ren https://orcid.org/0000-0002-6109-4085

REFERENCES

1. Dong, X., et al.: Time-varying formation control for unmanned aerial vehicles: Theories and applications. IEEE Trans. Control Syst. Technol. 23(1), 340–348 (2014)
2. Wu, S., Zheng, Z., Cai, K.Y.: Real-time path planning for unmanned aerial vehicles using behavior coordination and virtual goal. IET Control Theory Appl. 28(1), 131–136 (2011)
3. Li, Z., et al.: Consensus of multiagent systems and synchronization of complex networks: A unified viewpoint. IEEE Trans. Circuits Syst. 57(1), 213–224 (2010)
4. Xu, J. et al.: Consensus of multi-agent systems with general linear dynamics via dynamic output feedback control. IET Control Theory Appl. 7(1), 108–115 (2013)
5. Li, D., et al.: Cooperative circumnavigation control of networked microsatellites. IEEE Trans. Cybern. 50(10), 4550–4555 (2020)
6. Li, D., et al.: Distributed attitude coordinated control of multiple spacecraft with attitude constraints. IEEE Trans. Aerosp. Electron. Syst. 54(5), 2233–2245 (2018)
7. Mirzaei, M., Meskin, N., Abdollahi, F.: Robust consensus of autonomous underactuated surface vessels. IET Control Theory Appl. 11(4), 486–494 (2016)
8. Ren, W., Beard, R.W.: Consensus seeking in multiagent systems under dynamically changing interaction topologies. IEEE Trans. Autom. Control 50(5), 655–661 (2005)
9. Olfati-Saber, R., Murray, R.M.: Consensus problems in networks of agents with switching topology and time-delays. IEEE Trans. Autom. Control 49(9), 1520–1533 (2004)
10. Mu, X., Wu, X.: Tracking consensus for stochastic hybrid multi-agent systems with partly unknown transition rates via sliding mode control. IET Control Theory Appl. 14(8), 1091–1103 (2020)
11. Han, W., et al.: A simple approach to distributed observer design for linear systems. IEEE Trans. Autom. Control 64(1), 329–336 (2019)
12. Xiao, F., Wang, L.: Asynchronous consensus in continuous-time multi-agent systems with switching topology and time-varying delays. IEEE Trans. Autom. Control 53(8), 1804–1816 (2008)
13. Li, Z., et al.: Designing fully distributed consensus protocols for linear multi-agent systems with directed graphs. IEEE Trans. Autom. Control 60(4), 1152–1157 (2014)
14. Ren, W.: Consensus strategies for cooperative control of vehicle formations. IET Control Theory Appl. 1(2), 505–512 (2007)
15. Dong, X., et al.: Formation control for high-order linear time-invariant multiagent systems with time delays. IEEE Trans. Control Network Syst. 1(3), 232–240 (2014)
16. Xiao, F., et al.: Finite-time formation control for multi-agent systems. Automatica 45(11), 2605–2611 (2009)
17. Thunberg, J., Goncalves, J., Hu, X.: Consensus and formation control on SE (3) for switching topologies. Autom. 66, 109–121 (2016)
18. Wang, Y., et al.: Distributed control of nonlinear multiagent systems with unknown and nonidentical control directions via event-triggered communication. IEEE Trans. Cybern. 50(5), 1820–1832 (2019)
19. Cao, Y., Ren, W., Egerstedt, M.: Distributed containment control with multiple stationary or dynamic leaders in fixed and switching directed networks. Autom. 48(8), 1586–1597 (2012)
20. Haghshenas, H., Badamchizadeh, M.A., Baradarannia, M.: Containment control of heterogeneous linear multi-agent systems. Autom. 54, 210–216 (2015)
21. Yuan, C., Zeng, W.: Output containment control of heterogeneous multi-agent systems with leaders of bounded inputs: An adaptive finite-time observer approach. J. Franklin Inst. 356(6), 3419–3442 (2019)
22. Li, K., Mu, X.: Containment control of stochastic multiagent systems with semi-Markovian switching topologies. Int. J. Robust Nonlinear Control 29(14), 4943–4955 (2019)
23. Wang, Y., et al.: Output formation-containment of interacted heterogeneous linear systems by distributed hybrid active control. Autom. 93, 26–32 (2018)
24. Chen, L., et al.: Adaptive cooperative formation-containment control for networked Euler-Lagrange systems without using relative velocity information. IET Control Theory Appl. 11(9), 1450–1458 (2017)

25. Gong, X. et al.: Necessary and sufficient conditions of formation-containment control of high-order multi-agent systems with observer-type protocols. IEEE Trans. Cybern. https://doi.org/10.1109/TCYB.2020.3037133.
26. Li, D., et al.: Two-layer distributed formation-containment control of multiple Euler-Lagrange systems by output feedback. IEEE Trans. Cybern. 49(2), 675–687 (2018)
27. Ren, W.: Consensus based formation control strategies for multi-vehicle systems. In: 2006 American Control Conference, 2006, pp. 6
28. Lu, X., Austin, F., Chen, S.: Formation control for second-order multi-agent systems with time-varying delays under directed topology. Commun. Nonlinear Sci Numer. Simul. 17(3), 1382–1391 (2012)
29. Dong, X., et al.: Formation control for high-order linear time-invariant multiagent systems with time delays. IEEE Trans. Control Network Syst. 1(3), 232–240 (2014)
30. Dong, X., Hu, G.: Time-varying formation control for general linear multi-agent systems with switching directed topologies. Automatica 73, 47–55 (2016)
31. Xue, D., et al.: Formation control of multi-agent systems with stochastic switching topology and time-varying communication delays. IET Control Theory Appl. 7(13), 1689–1698 (2013)
32. Xiao, W., et al.: Time-varying formation control for time-delayed multi-agent systems with general linear dynamics and switching topologies. Unmanned Syst. 7(01), 3–13 (2019)
33. Ji, M., et al.: Containment control in mobile networks. IEEE Trans. Autom. Control 53(8), 1972–1975 (2008)
34. Wang, D., et al.: Cooperative containment control of multiagent systems based on follower observers with time delay. IEEE Trans. Syst., Man, Cybernetics: Syst. 47(1), 13–23 (2016)
35. Liu, S., Xie, L., Zhang, H.: Containment control of multiagent systems by exploiting the control inputs of neighbors. Int. J. Robust Nonlinear Control 24(17), 2803–2818 (2014)
36. Su, H., Chen, M.Z.: Multi-agent containment control with input saturation on switching topologies. IET Control Theory Appl. 9(3), 399–409 (2015)
37. Lou, Y., Hong, Y.: Target containment control of multi-agent systems with random switching interconnection topologies. Automatica 48(5), 879–885 (2012)
38. Dong, X., et al.: Formation-containment analysis and design for high-order linear time-invariant swarm systems. Int. J. Robust Nonlinear Control 25(17), 3439–3456 (2015)
39. Dong, X., et al.: Theory and experiment on formation-containment control of multiple multirotor unmanned aerial vehicle systems. IEEE Trans. Autom. Sci. Eng. 16(1), 229–240 (2018)
40. Dong, X., et al.: Formation-containment control for high-order linear time-invariant multi-agent systems with time delays. J. Franklin Inst. 352(9), 3564–3584 (2015)
41. Han, L., et al.: Formation-containment control for second-order multi-agent systems with time-varying delays. Neurocomputing 218, 439–447 (2016)
42. Han, T., et al.: Distributed three-dimensional formation containment control of multiple unmanned aerial vehicle systems. Asian J. Control 19(3), 1103–1113 (2017)
43. Zhang, X., et al.: Delay-dependent stabilization of linear systems with time-varying state and input delays. Automatica 41(8), 1405–1412 (2005)
44. Dong, X., Hu, G.: Time-varying formation tracking for linear multiagent systems with multiple leaders. IEEE Trans. Autom. Control 62(7), 3658–3664 (2017)
45. Han, L., Xie, Y., Li, X.: Time-varying group formation tracking control for second-order multi-agent systems with communication delays and multiple leaders. J. Franklin Inst. 357(14), 9761–9780 (2020)
46. Dong, X., Tan, Q., Li, Q.: Necessary and sufficient conditions for average formation tracking of second-order multi-agent systems with multiple leaders. J. Franklin Inst. 354(2), 611–626 (2017)
47. Hua, Y. et al.: Containment control of fully heterogeneous linear multi-agent systems with switching topologies. In: 15th International Conference on Control, Automation, Robotics and Vision (ICARCV), Singapore, 2018, pp. 538–543
48. Su, Y., Huang, J.: Cooperative output regulation of linear multi-agent systems. IEEE Trans. Autom. Control 57(4), 1062–1066 (2012)
49. Seyboth, G.S., Ren, W., Allgower, F.: Cooperative control of linear multi-agent systems via distributed output regulation and transient synchronization. Automatica 68, 132–139 (2016)
50. Lu, M., Liu, L.: Distributed feedforward approach to cooperative output regulation subject to communication delays and switching networks. IEEE Trans. Autom. Control 62(4), 1999–2005 (2017)
51. Hu, W., Liu, L., Feng, G.: Event-triggered cooperative output regulation of linear multiagent systems under jointly connected topologies. IEEE Trans. Autom. Control 64(3), 1317–1322 (2019)
52. Khalil, H.K.: Nonlinear systems. Prentice Hall, NJ (2002)

How to cite this article: Zhou S, Dong X, Hua Y, Yu J, Ren Z. Predefined formation-containment control of high-order multi-agent systems under communication delays and switching topologies. IET Control Theory Appl. 2021;15:1661–1672. https://doi.org/10.1049/cth2.12150